

# Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "6 Hyperbolic functions\6.7  
Miscellaneous"

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Test results for the 1059 problems in "6.7.1 Hyperbolic functions.m"

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \text{Cosh}[a + b x] \text{Sinh}[a + b x] dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\text{Sinh}[a + b x]^2}{2 b}$$

Result (type 3, 37 leaves):

$$\frac{1}{2} \left( \frac{\text{Cosh}[2 a] \text{Cosh}[2 b x]}{2 b} + \frac{\text{Sinh}[2 a] \text{Sinh}[2 b x]}{2 b} \right)$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[a + b x] \text{Sech}[a + b x] dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{\text{Log}[\text{Tanh}[a + b x]]}{b}$$

Result (type 3, 31 leaves):

$$2 \left( -\frac{\text{Log}[\text{Cosh}[a + b x]]}{2 b} + \frac{\text{Log}[\text{Sinh}[a + b x]]}{2 b} \right)$$

### Problem 29: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[a + b x]^2 \text{Sech}[a + b x] dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$-\frac{\text{ArcTan}[\text{Sinh}[a + b x]]}{b} - \frac{\text{Csch}[a + b x]}{b}$$

Result (type 3, 51 leaves):

$$-\frac{2 \text{ArcTan}[\text{Tanh}[\frac{1}{2}(a + b x)]]}{b} - \frac{\text{Coth}[\frac{1}{2}(a + b x)]}{2b} + \frac{\text{Tanh}[\frac{1}{2}(a + b x)]}{2b}$$

### Problem 39: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[a + b x]^4 \text{Sech}[a + b x] dx$$

Optimal (type 3, 37 leaves, 4 steps):

$$\frac{\text{ArcTan}[\text{Sinh}[a + b x]]}{b} + \frac{\text{Csch}[a + b x]}{b} - \frac{\text{Csch}[a + b x]^3}{3b}$$

Result (type 3, 109 leaves):

$$\frac{2 \text{ArcTan}[\text{Tanh}[\frac{1}{2}(a + b x)]]}{b} + \frac{7 \text{Coth}[\frac{1}{2}(a + b x)]}{12b} - \frac{\text{Coth}[\frac{1}{2}(a + b x)] \text{Csch}[\frac{1}{2}(a + b x)]^2}{24b} - \frac{7 \text{Tanh}[\frac{1}{2}(a + b x)]}{12b} - \frac{\text{Sech}[\frac{1}{2}(a + b x)]^2 \text{Tanh}[\frac{1}{2}(a + b x)]}{24b}$$

### Problem 49: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Sinh}[a + b x]^{7/2}}{\text{Cosh}[a + b x]^{7/2}} dx$$

Optimal (type 3, 106 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\text{Cosh}[a + b x]}}{\sqrt{\text{Sinh}[a + b x]}}\right]}{b} + \frac{\text{ArcTanh}\left[\frac{\sqrt{\text{Cosh}[a + b x]}}{\sqrt{\text{Sinh}[a + b x]}}\right]}{b} - \frac{2\sqrt{\text{Sinh}[a + b x]}}{b\sqrt{\text{Cosh}[a + b x]}} - \frac{2\text{Sinh}[a + b x]^{5/2}}{5b\text{Cosh}[a + b x]^{5/2}}$$

Result (type 5, 98 leaves):

$$\left( 2 \operatorname{Sinh}[a + b x]^{5/2} \left( 5 \operatorname{Cosh}[a + b x]^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Cosh}[a + b x]^2\right] + 3 (2 + 3 \operatorname{Cosh}[2(a + b x)]) (-\operatorname{Sinh}[a + b x]^2)^{1/4} \right) \right) / \left( 15 b \operatorname{Cosh}[a + b x]^{5/2} (-\operatorname{Sinh}[a + b x]^2)^{5/4} \right)$$

**Problem 50: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sinh}[a + b x]^{5/2}}{\operatorname{Cosh}[a + b x]^{5/2}} dx$$

Optimal (type 3, 81 leaves, 5 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{\operatorname{Sinh}[a + b x]}}{\sqrt{\operatorname{Cosh}[a + b x]}}\right]}{b} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{Sinh}[a + b x]}}{\sqrt{\operatorname{Cosh}[a + b x]}}\right]}{b} - \frac{2 \operatorname{Sinh}[a + b x]^{3/2}}{3 b \operatorname{Cosh}[a + b x]^{3/2}}$$

Result (type 5, 85 leaves):

$$-\frac{2 \operatorname{Sinh}[a + b x]^{3/2}}{3 b \operatorname{Cosh}[a + b x]^{3/2}} - \frac{2 \sqrt{\operatorname{Cosh}[a + b x]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Cosh}[a + b x]^2\right] \operatorname{Sinh}[a + b x]^{3/2}}{b (-\operatorname{Sinh}[a + b x]^2)^{3/4}}$$

**Problem 51: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sinh}[a + b x]^{3/2}}{\operatorname{Cosh}[a + b x]^{3/2}} dx$$

Optimal (type 3, 79 leaves, 5 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{\operatorname{Cosh}[a + b x]}}{\sqrt{\operatorname{Sinh}[a + b x]}}\right]}{b} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{Cosh}[a + b x]}}{\sqrt{\operatorname{Sinh}[a + b x]}}\right]}{b} - \frac{2 \sqrt{\operatorname{Sinh}[a + b x]}}{b \sqrt{\operatorname{Cosh}[a + b x]}}$$

Result (type 5, 85 leaves):

$$-\frac{2 \sqrt{\operatorname{Sinh}[a + b x]}}{b \sqrt{\operatorname{Cosh}[a + b x]}} - \frac{2 \operatorname{Cosh}[a + b x]^{3/2} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Cosh}[a + b x]^2\right] \sqrt{\operatorname{Sinh}[a + b x]}}{3 b (-\operatorname{Sinh}[a + b x]^2)^{1/4}}$$

**Problem 52: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\operatorname{Sinh}[a + b x]}}{\sqrt{\operatorname{Cosh}[a + b x]}} dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\text{Sinh}[a+bx]}}{\sqrt{\text{Cosh}[a+bx]}}\right]}{b} + \frac{\text{ArcTanh}\left[\frac{\sqrt{\text{Sinh}[a+bx]}}{\sqrt{\text{Cosh}[a+bx]}}\right]}{b}$$

Result (type 5, 57 leaves):

$$-\frac{2\sqrt{\text{Cosh}[a+bx]}\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \text{Cosh}[a+bx]^2\right]\text{Sinh}[a+bx]^{3/2}}{b\left(-\text{Sinh}[a+bx]^2\right)^{3/4}}$$

**Problem 53: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\text{Cosh}[a+bx]}}{\sqrt{\text{Sinh}[a+bx]}} dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\text{Cosh}[a+bx]}}{\sqrt{\text{Sinh}[a+bx]}}\right]}{b} + \frac{\text{ArcTanh}\left[\frac{\sqrt{\text{Cosh}[a+bx]}}{\sqrt{\text{Sinh}[a+bx]}}\right]}{b}$$

Result (type 5, 59 leaves):

$$-\frac{2\text{Cosh}[a+bx]^{3/2}\text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \text{Cosh}[a+bx]^2\right]\sqrt{\text{Sinh}[a+bx]}}{3b\left(-\text{Sinh}[a+bx]^2\right)^{1/4}}$$

**Problem 54: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{Cosh}[a+bx]^{3/2}}{\text{Sinh}[a+bx]^{3/2}} dx$$

Optimal (type 3, 79 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\text{Sinh}[a+bx]}}{\sqrt{\text{Cosh}[a+bx]}}\right]}{b} + \frac{\text{ArcTanh}\left[\frac{\sqrt{\text{Sinh}[a+bx]}}{\sqrt{\text{Cosh}[a+bx]}}\right]}{b} - \frac{2\sqrt{\text{Cosh}[a+bx]}}{b\sqrt{\text{Sinh}[a+bx]}}$$

Result (type 5, 83 leaves):

$$-\frac{2\sqrt{\text{Cosh}[a+bx]}}{b\sqrt{\text{Sinh}[a+bx]}} - \frac{2\sqrt{\text{Cosh}[a+bx]}\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \text{Cosh}[a+bx]^2\right]\text{Sinh}[a+bx]^{3/2}}{b\left(-\text{Sinh}[a+bx]^2\right)^{3/4}}$$

### Problem 55: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Cosh}[a + b x]^{5/2}}{\text{Sinh}[a + b x]^{5/2}} dx$$

Optimal (type 3, 81 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\text{Cosh}[a+bx]}}{\sqrt{\text{Sinh}[a+bx]}}\right]}{b} + \frac{\text{ArcTanh}\left[\frac{\sqrt{\text{Cosh}[a+bx]}}{\sqrt{\text{Sinh}[a+bx]}}\right]}{b} - \frac{2 \text{Cosh}[a + b x]^{3/2}}{3 b \text{Sinh}[a + b x]^{3/2}}$$

Result (type 5, 83 leaves):

$$\frac{1}{3 b (-\text{Sinh}[a + b x]^2)^{5/4}} 2 \text{Cosh}[a + b x]^{3/2} \sqrt{\text{Sinh}[a + b x]} \left( \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \text{Cosh}[a + b x]^2\right] \text{Sinh}[a + b x]^2 + (-\text{Sinh}[a + b x]^2)^{1/4} \right)$$

### Problem 56: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Cosh}[a + b x]^{7/2}}{\text{Sinh}[a + b x]^{7/2}} dx$$

Optimal (type 3, 106 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\text{Sinh}[a+bx]}}{\sqrt{\text{Cosh}[a+bx]}}\right]}{b} + \frac{\text{ArcTanh}\left[\frac{\sqrt{\text{Sinh}[a+bx]}}{\sqrt{\text{Cosh}[a+bx]}}\right]}{b} - \frac{2 \text{Cosh}[a + b x]^{5/2}}{5 b \text{Sinh}[a + b x]^{5/2}} - \frac{2 \sqrt{\text{Cosh}[a + b x]}}{b \sqrt{\text{Sinh}[a + b x]}}$$

Result (type 5, 97 leaves):

$$\left( 2 \sqrt{\text{Cosh}[a + b x]} \left( 5 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \text{Cosh}[a + b x]^2\right] \text{Sinh}[a + b x]^4 + (-\text{Sinh}[a + b x]^2)^{3/4} (1 + 6 \text{Sinh}[a + b x]^2) \right) \right) / \left( 5 b \sqrt{\text{Sinh}[a + b x]} (-\text{Sinh}[a + b x]^2)^{7/4} \right)$$

### Problem 57: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Sinh}[a + b x]^{7/3}}{\text{Cosh}[a + b x]^{7/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$-\frac{\sqrt{3} \text{ArcTan}\left[\frac{1 + \frac{2 \text{Sinh}[a+bx]^{2/3}}{\text{Cosh}[a+bx]^{2/3}}}{\sqrt{3}}\right]}{2 b} - \frac{\text{Log}\left[1 - \frac{\text{Sinh}[a+bx]^{2/3}}{\text{Cosh}[a+bx]^{2/3}}\right]}{2 b} + \frac{\text{Log}\left[1 + \frac{\text{Sinh}[a+bx]^{2/3}}{\text{Cosh}[a+bx]^{2/3}} + \frac{\text{Sinh}[a+bx]^{4/3}}{\text{Cosh}[a+bx]^{4/3}}\right]}{4 b} - \frac{3 \text{Sinh}[a + b x]^{4/3}}{4 b \text{Cosh}[a + b x]^{4/3}}$$

Result (type 5, 80 leaves):

$$\frac{3 \left( -\operatorname{Sinh}[a + b x]^2 + 2 \operatorname{Cosh}[a + b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \operatorname{Cosh}[a + b x]^2\right] (-\operatorname{Sinh}[a + b x]^2)^{1/3} \right)}{4 b \operatorname{Cosh}[a + b x]^{4/3} \operatorname{Sinh}[a + b x]^{2/3}}$$

Problem 58: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{Sinh}[a + b x]^{5/3}}{\operatorname{Cosh}[a + b x]^{5/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Cosh}[a + b x]^{2/3}}{\operatorname{Sinh}[a + b x]^{2/3}}}{\sqrt{3}}\right]}{2 b} - \frac{\operatorname{Log}\left[1 - \frac{\operatorname{Cosh}[a + b x]^{2/3}}{\operatorname{Sinh}[a + b x]^{2/3}}\right]}{2 b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Cosh}[a + b x]^{4/3}}{\operatorname{Sinh}[a + b x]^{4/3}} + \frac{\operatorname{Cosh}[a + b x]^{2/3}}{\operatorname{Sinh}[a + b x]^{2/3}}\right]}{4 b} - \frac{3 \operatorname{Sinh}[a + b x]^{2/3}}{2 b \operatorname{Cosh}[a + b x]^{2/3}}$$

Result (type 5, 87 leaves):

$$-\frac{3 \operatorname{Sinh}[a + b x]^{2/3}}{2 b \operatorname{Cosh}[a + b x]^{2/3}} - \frac{3 \operatorname{Cosh}[a + b x]^{4/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \operatorname{Cosh}[a + b x]^2\right] \operatorname{Sinh}[a + b x]^{2/3}}{4 b (-\operatorname{Sinh}[a + b x]^2)^{1/3}}$$

Problem 59: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{Sinh}[a + b x]^{4/3}}{\operatorname{Cosh}[a + b x]^{4/3}} dx$$

Optimal (type 3, 243 leaves, 12 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \operatorname{Cosh}[a + b x]^{1/3}}{\operatorname{Sinh}[a + b x]^{1/3}}}{\sqrt{3}}\right]}{2 b} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Cosh}[a + b x]^{1/3}}{\operatorname{Sinh}[a + b x]^{1/3}}}{\sqrt{3}}\right]}{2 b} + \frac{\operatorname{ArcTanh}\left[\frac{\operatorname{Cosh}[a + b x]^{1/3}}{\operatorname{Sinh}[a + b x]^{1/3}}\right]}{b} - \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Cosh}[a + b x]^{2/3}}{\operatorname{Sinh}[a + b x]^{2/3}} - \frac{\operatorname{Cosh}[a + b x]^{1/3}}{\operatorname{Sinh}[a + b x]^{1/3}}\right]}{4 b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Cosh}[a + b x]^{2/3}}{\operatorname{Sinh}[a + b x]^{2/3}} + \frac{\operatorname{Cosh}[a + b x]^{1/3}}{\operatorname{Sinh}[a + b x]^{1/3}}\right]}{4 b} - \frac{3 \operatorname{Sinh}[a + b x]^{1/3}}{b \operatorname{Cosh}[a + b x]^{1/3}}$$

Result (type 5, 85 leaves):

$$-\frac{3 \operatorname{Sinh}[a + b x]^{1/3}}{b \operatorname{Cosh}[a + b x]^{1/3}} - \frac{3 \operatorname{Cosh}[a + b x]^{5/3} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \operatorname{Cosh}[a + b x]^2\right] \operatorname{Sinh}[a + b x]^{1/3}}{5 b (-\operatorname{Sinh}[a + b x]^2)^{1/6}}$$

### Problem 60: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Sinh}[a + b x]^{2/3}}{\text{Cosh}[a + b x]^{2/3}} dx$$

Optimal (type 3, 218 leaves, 11 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \operatorname{Sinh}[a + b x]^{1/3}}{\operatorname{Cosh}[a + b x]^{1/3}}}{\sqrt{3}}\right]}{2 b} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Sinh}[a + b x]^{1/3}}{\operatorname{Cosh}[a + b x]^{1/3}}}{\sqrt{3}}\right]}{2 b} +$$

$$\frac{\operatorname{ArcTanh}\left[\frac{\operatorname{Sinh}[a + b x]^{1/3}}{\operatorname{Cosh}[a + b x]^{1/3}}\right]}{b} - \frac{\operatorname{Log}\left[1 - \frac{\operatorname{Sinh}[a + b x]^{1/3}}{\operatorname{Cosh}[a + b x]^{1/3}} + \frac{\operatorname{Sinh}[a + b x]^{2/3}}{\operatorname{Cosh}[a + b x]^{2/3}}\right]}{4 b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Sinh}[a + b x]^{1/3}}{\operatorname{Cosh}[a + b x]^{1/3}} + \frac{\operatorname{Sinh}[a + b x]^{2/3}}{\operatorname{Cosh}[a + b x]^{2/3}}\right]}{4 b}$$

Result (type 5, 57 leaves):

$$\frac{3 \operatorname{Cosh}[a + b x]^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \operatorname{Cosh}[a + b x]^2\right] \operatorname{Sinh}[a + b x]^{5/3}}{b \left(-\operatorname{Sinh}[a + b x]^2\right)^{5/6}}$$

### Problem 61: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Sinh}[a + b x]^{1/3}}{\text{Cosh}[a + b x]^{1/3}} dx$$

Optimal (type 3, 128 leaves, 8 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Sinh}[a + b x]^{2/3}}{\operatorname{Cosh}[a + b x]^{2/3}}}{\sqrt{3}}\right]}{2 b} - \frac{\operatorname{Log}\left[1 - \frac{\operatorname{Sinh}[a + b x]^{2/3}}{\operatorname{Cosh}[a + b x]^{2/3}}\right]}{2 b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Sinh}[a + b x]^{2/3}}{\operatorname{Cosh}[a + b x]^{2/3}} + \frac{\operatorname{Sinh}[a + b x]^{4/3}}{\operatorname{Cosh}[a + b x]^{4/3}}\right]}{4 b}$$

Result (type 5, 59 leaves):

$$\frac{3 \operatorname{Cosh}[a + b x]^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \operatorname{Cosh}[a + b x]^2\right] \operatorname{Sinh}[a + b x]^{4/3}}{2 b \left(-\operatorname{Sinh}[a + b x]^2\right)^{2/3}}$$

### Problem 62: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Cosh}[a + b x]^{1/3}}{\text{Sinh}[a + b x]^{1/3}} dx$$

Optimal (type 3, 128 leaves, 8 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Cosh}[a+bx]^{2/3}}{\operatorname{Sinh}[a+bx]^{2/3}}}{\sqrt{3}}\right]}{2b} - \frac{\operatorname{Log}\left[1 - \frac{\operatorname{Cosh}[a+bx]^{2/3}}{\operatorname{Sinh}[a+bx]^{2/3}}\right]}{2b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Cosh}[a+bx]^{4/3}}{\operatorname{Sinh}[a+bx]^{4/3}} + \frac{\operatorname{Cosh}[a+bx]^{2/3}}{\operatorname{Sinh}[a+bx]^{2/3}}\right]}{4b}$$

Result (type 5, 59 leaves):

$$-\frac{3 \operatorname{Cosh}[a+bx]^{4/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \operatorname{Cosh}[a+bx]^2\right] \operatorname{Sinh}[a+bx]^{2/3}}{4b \left(-\operatorname{Sinh}[a+bx]^2\right)^{1/3}}$$

**Problem 63: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Cosh}[a+bx]^{2/3}}{\operatorname{Sinh}[a+bx]^{2/3}} dx$$

Optimal (type 3, 218 leaves, 11 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \operatorname{Cosh}[a+bx]^{1/3}}{\operatorname{Sinh}[a+bx]^{1/3}}}{\sqrt{3}}\right]}{2b} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Cosh}[a+bx]^{1/3}}{\operatorname{Sinh}[a+bx]^{1/3}}}{\sqrt{3}}\right]}{2b} +$$

$$\frac{\operatorname{ArcTanh}\left[\frac{\operatorname{Cosh}[a+bx]^{1/3}}{\operatorname{Sinh}[a+bx]^{1/3}}\right]}{b} - \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Cosh}[a+bx]^{2/3}}{\operatorname{Sinh}[a+bx]^{2/3}} - \frac{\operatorname{Cosh}[a+bx]^{1/3}}{\operatorname{Sinh}[a+bx]^{1/3}}\right]}{4b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Cosh}[a+bx]^{2/3}}{\operatorname{Sinh}[a+bx]^{2/3}} + \frac{\operatorname{Cosh}[a+bx]^{1/3}}{\operatorname{Sinh}[a+bx]^{1/3}}\right]}{4b}$$

Result (type 5, 59 leaves):

$$-\frac{3 \operatorname{Cosh}[a+bx]^{5/3} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \operatorname{Cosh}[a+bx]^2\right] \operatorname{Sinh}[a+bx]^{1/3}}{5b \left(-\operatorname{Sinh}[a+bx]^2\right)^{1/6}}$$

**Problem 64: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Cosh}[a+bx]^{4/3}}{\operatorname{Sinh}[a+bx]^{4/3}} dx$$

Optimal (type 3, 243 leaves, 12 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \operatorname{Sinh}[a+bx]^{1/3}}{\operatorname{Cosh}[a+bx]^{1/3}}}{\sqrt{3}}\right]}{2b} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Sinh}[a+bx]^{1/3}}{\operatorname{Cosh}[a+bx]^{1/3}}}{\sqrt{3}}\right]}{2b} + \frac{\operatorname{ArcTanh}\left[\frac{\operatorname{Sinh}[a+bx]^{1/3}}{\operatorname{Cosh}[a+bx]^{1/3}}\right]}{b} -$$

$$\frac{\operatorname{Log}\left[1 - \frac{\operatorname{Sinh}[a+bx]^{1/3}}{\operatorname{Cosh}[a+bx]^{1/3}} + \frac{\operatorname{Sinh}[a+bx]^{2/3}}{\operatorname{Cosh}[a+bx]^{2/3}}\right]}{4b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Sinh}[a+bx]^{1/3}}{\operatorname{Cosh}[a+bx]^{1/3}} + \frac{\operatorname{Sinh}[a+bx]^{2/3}}{\operatorname{Cosh}[a+bx]^{2/3}}\right]}{4b} - \frac{3 \operatorname{Cosh}[a+bx]^{1/3}}{b \operatorname{Sinh}[a+bx]^{1/3}}$$



Result (type 5, 83 leaves):

$$\frac{3 \operatorname{Cosh}[a + b x]^{1/3} \operatorname{Cosh}[a + b x]^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \operatorname{Cosh}[a + b x]^2\right] \operatorname{Sinh}[a + b x]^{5/3}}{b \operatorname{Sinh}[a + b x]^{1/3} b \left(-\operatorname{Sinh}[a + b x]^2\right)^{5/6}}$$

Problem 65: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{Cosh}[a + b x]^{5/3}}{\operatorname{Sinh}[a + b x]^{5/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Sinh}[a + b x]^{2/3}}{\operatorname{Cosh}[a + b x]^{2/3}}}{\sqrt{3}}\right]}{2 b} - \frac{\operatorname{Log}\left[1 - \frac{\operatorname{Sinh}[a + b x]^{2/3}}{\operatorname{Cosh}[a + b x]^{2/3}}\right]}{2 b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Sinh}[a + b x]^{2/3}}{\operatorname{Cosh}[a + b x]^{2/3}} + \frac{\operatorname{Sinh}[a + b x]^{4/3}}{\operatorname{Cosh}[a + b x]^{4/3}}\right]}{4 b} - \frac{3 \operatorname{Cosh}[a + b x]^{2/3}}{2 b \operatorname{Sinh}[a + b x]^{2/3}}$$

Result (type 5, 83 leaves):

$$\frac{1}{2 b \left(-\operatorname{Sinh}[a + b x]^2\right)^{5/3}} 3 \operatorname{Cosh}[a + b x]^{2/3} \operatorname{Sinh}[a + b x]^{4/3} \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \operatorname{Cosh}[a + b x]^2\right] \operatorname{Sinh}[a + b x]^2 + \left(-\operatorname{Sinh}[a + b x]^2\right)^{2/3}\right)$$

Problem 66: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{Cosh}[a + b x]^{7/3}}{\operatorname{Sinh}[a + b x]^{7/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \operatorname{Cosh}[a + b x]^{2/3}}{\operatorname{Sinh}[a + b x]^{2/3}}}{\sqrt{3}}\right]}{2 b} - \frac{\operatorname{Log}\left[1 - \frac{\operatorname{Cosh}[a + b x]^{2/3}}{\operatorname{Sinh}[a + b x]^{2/3}}\right]}{2 b} + \frac{\operatorname{Log}\left[1 + \frac{\operatorname{Cosh}[a + b x]^{4/3}}{\operatorname{Sinh}[a + b x]^{4/3}} + \frac{\operatorname{Cosh}[a + b x]^{2/3}}{\operatorname{Sinh}[a + b x]^{2/3}}\right]}{4 b} - \frac{3 \operatorname{Cosh}[a + b x]^{4/3}}{4 b \operatorname{Sinh}[a + b x]^{4/3}}$$

Result (type 5, 83 leaves):

$$\frac{1}{4 b \left(-\operatorname{Sinh}[a + b x]^2\right)^{4/3}} 3 \operatorname{Cosh}[a + b x]^{4/3} \operatorname{Sinh}[a + b x]^{2/3} \left(\operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \operatorname{Cosh}[a + b x]^2\right] \operatorname{Sinh}[a + b x]^2 + \left(-\operatorname{Sinh}[a + b x]^2\right)^{1/3}\right)$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[x]^8 \operatorname{Tanh}[x]^6 dx$$

Optimal (type 3, 33 leaves, 3 steps):

$$\frac{\text{Tanh}[x]^7}{7} - \frac{\text{Tanh}[x]^9}{3} + \frac{3 \text{Tanh}[x]^{11}}{11} - \frac{\text{Tanh}[x]^{13}}{13}$$

Result (type 3, 67 leaves):

$$\frac{16 \text{Tanh}[x]}{3003} + \frac{8 \text{Sech}[x]^2 \text{Tanh}[x]}{3003} + \frac{2 \text{Sech}[x]^4 \text{Tanh}[x]}{1001} + \frac{5 \text{Sech}[x]^6 \text{Tanh}[x]}{3003} - \frac{53}{429} \text{Sech}[x]^8 \text{Tanh}[x] + \frac{27}{143} \text{Sech}[x]^{10} \text{Tanh}[x] - \frac{1}{13} \text{Sech}[x]^{12} \text{Tanh}[x]$$

**Problem 102: Result more than twice size of optimal antiderivative.**

$$\int \text{Cosh}[a + b x] \text{Coth}[a + b x]^2 dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$-\frac{\text{Csch}[a + b x]}{b} + \frac{\text{Sinh}[a + b x]}{b}$$

Result (type 3, 45 leaves):

$$-\frac{\text{Coth}\left[\frac{1}{2}(a + b x)\right]}{2b} + \frac{\text{Sinh}[a + b x]}{b} + \frac{\text{Tanh}\left[\frac{1}{2}(a + b x)\right]}{2b}$$

**Problem 104: Result more than twice size of optimal antiderivative.**

$$\int \text{Cosh}[a + b x] \text{Coth}[a + b x]^4 dx$$

Optimal (type 3, 37 leaves, 3 steps):

$$-\frac{2 \text{Csch}[a + b x]}{b} - \frac{\text{Csch}[a + b x]^3}{3b} + \frac{\text{Sinh}[a + b x]}{b}$$

Result (type 3, 103 leaves):

$$-\frac{11 \text{Coth}\left[\frac{1}{2}(a + b x)\right]}{12b} - \frac{\text{Coth}\left[\frac{1}{2}(a + b x)\right] \text{Csch}\left[\frac{1}{2}(a + b x)\right]^2}{24b} + \frac{\text{Sinh}[a + b x]}{b} + \frac{11 \text{Tanh}\left[\frac{1}{2}(a + b x)\right]}{12b} - \frac{\text{Sech}\left[\frac{1}{2}(a + b x)\right]^2 \text{Tanh}\left[\frac{1}{2}(a + b x)\right]}{24b}$$

**Problem 118: Result more than twice size of optimal antiderivative.**

$$\int \text{Coth}[a + b x]^3 \text{Csch}[a + b x] dx$$

Optimal (type 3, 27 leaves, 2 steps):

$$\frac{\operatorname{Csch}[a + b x]}{b} - \frac{\operatorname{Csch}[a + b x]^3}{3 b}$$

Result (type 3, 93 leaves):

$$-\frac{5 \operatorname{Coth}\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\operatorname{Coth}\left[\frac{1}{2}(a + b x)\right] \operatorname{Csch}\left[\frac{1}{2}(a + b x)\right]^2}{24 b} + \frac{5 \operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\operatorname{Sech}\left[\frac{1}{2}(a + b x)\right]^2 \operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]}{24 b}$$

**Problem 119: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Coth}[a + b x]^3 \operatorname{Csch}[a + b x]^3 dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$\frac{\operatorname{Csch}[a + b x]^3}{3 b} - \frac{\operatorname{Csch}[a + b x]^5}{5 b}$$

Result (type 3, 151 leaves):

$$\frac{11 \operatorname{Coth}\left[\frac{1}{2}(a + b x)\right]}{240 b} - \frac{11 \operatorname{Coth}\left[\frac{1}{2}(a + b x)\right] \operatorname{Csch}\left[\frac{1}{2}(a + b x)\right]^2}{480 b} - \frac{\operatorname{Coth}\left[\frac{1}{2}(a + b x)\right] \operatorname{Csch}\left[\frac{1}{2}(a + b x)\right]^4}{160 b} - \frac{11 \operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]}{240 b} - \frac{11 \operatorname{Sech}\left[\frac{1}{2}(a + b x)\right]^2 \operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]}{480 b} + \frac{\operatorname{Sech}\left[\frac{1}{2}(a + b x)\right]^4 \operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]}{160 b}$$

**Problem 121: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Coth}[a + b x]^2 \operatorname{Csch}[a + b x] dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[a + b x]]}{2 b} - \frac{\operatorname{Coth}[a + b x] \operatorname{Csch}[a + b x]}{2 b}$$

Result (type 3, 75 leaves):

$$-\frac{\operatorname{Csch}\left[\frac{1}{2}(a + b x)\right]^2}{8 b} - \frac{\operatorname{Log}[\operatorname{Cosh}\left[\frac{1}{2}(a + b x)\right]]}{2 b} + \frac{\operatorname{Log}[\operatorname{Sinh}\left[\frac{1}{2}(a + b x)\right]]}{2 b} - \frac{\operatorname{Sech}\left[\frac{1}{2}(a + b x)\right]^2}{8 b}$$

### Problem 122: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[a + b x]^2 \text{Csch}[a + b x]^3 dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$\frac{\text{ArcTanh}[\text{Cosh}[a + b x]]}{8 b} - \frac{\text{Coth}[a + b x] \text{Csch}[a + b x]}{8 b} - \frac{\text{Coth}[a + b x] \text{Csch}[a + b x]^3}{4 b}$$

Result (type 3, 113 leaves):

$$-\frac{\text{Csch}\left[\frac{1}{2}(a + b x)\right]^2}{32 b} - \frac{\text{Csch}\left[\frac{1}{2}(a + b x)\right]^4}{64 b} + \frac{\text{Log}\left[\text{Cosh}\left[\frac{1}{2}(a + b x)\right]\right]}{8 b} - \frac{\text{Log}\left[\text{Sinh}\left[\frac{1}{2}(a + b x)\right]\right]}{8 b} - \frac{\text{Sech}\left[\frac{1}{2}(a + b x)\right]^2}{32 b} + \frac{\text{Sech}\left[\frac{1}{2}(a + b x)\right]^4}{64 b}$$

### Problem 123: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[a + b x]^4 \text{Csch}[a + b x] dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$-\frac{3 \text{ArcTanh}[\text{Cosh}[a + b x]]}{8 b} - \frac{3 \text{Coth}[a + b x] \text{Csch}[a + b x]}{8 b} - \frac{\text{Coth}[a + b x]^3 \text{Csch}[a + b x]}{4 b}$$

Result (type 3, 113 leaves):

$$-\frac{5 \text{Csch}\left[\frac{1}{2}(a + b x)\right]^2}{32 b} - \frac{\text{Csch}\left[\frac{1}{2}(a + b x)\right]^4}{64 b} - \frac{3 \text{Log}\left[\text{Cosh}\left[\frac{1}{2}(a + b x)\right]\right]}{8 b} + \frac{3 \text{Log}\left[\text{Sinh}\left[\frac{1}{2}(a + b x)\right]\right]}{8 b} - \frac{5 \text{Sech}\left[\frac{1}{2}(a + b x)\right]^2}{32 b} + \frac{\text{Sech}\left[\frac{1}{2}(a + b x)\right]^4}{64 b}$$

### Problem 127: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[x]^4 \text{Csch}[x]^3 dx$$

Optimal (type 3, 38 leaves, 4 steps):

$$\frac{1}{16} \text{ArcTanh}[\text{Cosh}[x]] - \frac{1}{16} \text{Coth}[x] \text{Csch}[x] - \frac{1}{8} \text{Coth}[x] \text{Csch}[x]^3 - \frac{1}{6} \text{Coth}[x]^3 \text{Csch}[x]^3$$

Result (type 3, 95 leaves):

$$-\frac{1}{64} \text{Csch}\left[\frac{x}{2}\right]^2 - \frac{1}{64} \text{Csch}\left[\frac{x}{2}\right]^4 - \frac{1}{384} \text{Csch}\left[\frac{x}{2}\right]^6 + \frac{1}{16} \text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] - \frac{1}{16} \text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right] - \frac{1}{64} \text{Sech}\left[\frac{x}{2}\right]^2 + \frac{1}{64} \text{Sech}\left[\frac{x}{2}\right]^4 - \frac{1}{384} \text{Sech}\left[\frac{x}{2}\right]^6$$

### Problem 129: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[6x]^5 \operatorname{Csch}[6x] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$-\frac{1}{6} \operatorname{Csch}[6x] - \frac{1}{9} \operatorname{Csch}[6x]^3 - \frac{1}{30} \operatorname{Csch}[6x]^5$$

Result (type 3, 73 leaves):

$$-\frac{89 \operatorname{Coth}[3x]}{1440} - \frac{31 \operatorname{Coth}[3x] \operatorname{Csch}[3x]^2}{2880} - \frac{1}{960} \operatorname{Coth}[3x] \operatorname{Csch}[3x]^4 + \frac{89 \operatorname{Tanh}[3x]}{1440} - \frac{31 \operatorname{Sech}[3x]^2 \operatorname{Tanh}[3x]}{2880} + \frac{1}{960} \operatorname{Sech}[3x]^4 \operatorname{Tanh}[3x]$$

### Problem 130: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[x]^7 \operatorname{Csch}[x]^3 dx$$

Optimal (type 3, 33 leaves, 3 steps):

$$-\frac{1}{3} \operatorname{Csch}[x]^3 - \frac{3 \operatorname{Csch}[x]^5}{5} - \frac{3 \operatorname{Csch}[x]^7}{7} - \frac{\operatorname{Csch}[x]^9}{9}$$

Result (type 3, 165 leaves):

$$\frac{1823 \operatorname{Coth}\left[\frac{x}{2}\right]}{80640} - \frac{1823 \operatorname{Coth}\left[\frac{x}{2}\right] \operatorname{Csch}\left[\frac{x}{2}\right]^2}{161280} - \frac{463 \operatorname{Coth}\left[\frac{x}{2}\right] \operatorname{Csch}\left[\frac{x}{2}\right]^4}{53760} - \frac{73 \operatorname{Coth}\left[\frac{x}{2}\right] \operatorname{Csch}\left[\frac{x}{2}\right]^6}{32256} - \frac{\operatorname{Coth}\left[\frac{x}{2}\right] \operatorname{Csch}\left[\frac{x}{2}\right]^8}{4608} - \frac{1823 \operatorname{Tanh}\left[\frac{x}{2}\right]}{80640} - \frac{1823 \operatorname{Sech}\left[\frac{x}{2}\right]^2 \operatorname{Tanh}\left[\frac{x}{2}\right]}{161280} + \frac{463 \operatorname{Sech}\left[\frac{x}{2}\right]^4 \operatorname{Tanh}\left[\frac{x}{2}\right]}{53760} - \frac{73 \operatorname{Sech}\left[\frac{x}{2}\right]^6 \operatorname{Tanh}\left[\frac{x}{2}\right]}{32256} + \frac{\operatorname{Sech}\left[\frac{x}{2}\right]^8 \operatorname{Tanh}\left[\frac{x}{2}\right]}{4608}$$

### Problem 143: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sinh}[a+bx] \operatorname{Tanh}[c+bx] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan}[\operatorname{Sinh}[c+bx]] \operatorname{Cosh}[a-c]}{b} + \frac{\operatorname{Sinh}[a+bx]}{b}$$

Result (type 3, 86 leaves):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])\left(\operatorname{Cosh}\left[\frac{bx}{2}\right]\operatorname{Sinh}[c]+\operatorname{Cosh}[c]\operatorname{Sinh}\left[\frac{bx}{2}\right]\right)}{\operatorname{Cosh}[c]\operatorname{Cosh}\left[\frac{bx}{2}\right]-\operatorname{Cosh}\left[\frac{bx}{2}\right]\operatorname{Sinh}[c]}\right]}{b} \operatorname{Cosh}[a-c] + \frac{\operatorname{Cosh}[bx]\operatorname{Sinh}[a]}{b} + \frac{\operatorname{Cosh}[a]\operatorname{Sinh}[bx]}{b}$$

**Problem 144: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sinh}[a+bx] \operatorname{Tanh}[c+bx]^2 dx$$

Optimal (type 3, 45 leaves, 6 steps):

$$\frac{\operatorname{Cosh}[a+bx]}{b} + \frac{\operatorname{Cosh}[a-c] \operatorname{Sech}[c+bx]}{b} - \frac{\operatorname{ArcTan}[\operatorname{Sinh}[c+bx]] \operatorname{Sinh}[a-c]}{b}$$

Result (type 3, 102 leaves):

$$\frac{\operatorname{Cosh}[a]\operatorname{Cosh}[bx]}{b} + \frac{\operatorname{Cosh}[a-c] \operatorname{Sech}[c+bx]}{b} - \frac{2 \operatorname{ArcTan}\left[\frac{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])\left(\operatorname{Cosh}\left[\frac{bx}{2}\right]\operatorname{Sinh}[c]+\operatorname{Cosh}[c]\operatorname{Sinh}\left[\frac{bx}{2}\right]\right)}{\operatorname{Cosh}[c]\operatorname{Cosh}\left[\frac{bx}{2}\right]-\operatorname{Cosh}\left[\frac{bx}{2}\right]\operatorname{Sinh}[c]}\right]}{b} \operatorname{Sinh}[a-c] + \frac{\operatorname{Sinh}[a]\operatorname{Sinh}[bx]}{b}$$

**Problem 146: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Coth}[c+bx] \operatorname{Sinh}[a+bx] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+bx]] \operatorname{Sinh}[a-c]}{b} + \frac{\operatorname{Sinh}[a+bx]}{b}$$

Result (type 3, 93 leaves):

$$\frac{\operatorname{Cosh}[bx] \operatorname{Sinh}[a]}{b} - \frac{2 i \operatorname{ArcTan}\left[\frac{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])\left(\operatorname{Cosh}[c]\operatorname{Cosh}\left[\frac{bx}{2}\right]+\operatorname{Sinh}[c]\operatorname{Sinh}\left[\frac{bx}{2}\right]\right)}{i \operatorname{Cosh}[c]\operatorname{Cosh}\left[\frac{bx}{2}\right]-i \operatorname{Cosh}\left[\frac{bx}{2}\right]\operatorname{Sinh}[c]}\right]}{b} \operatorname{Sinh}[a-c] + \frac{\operatorname{Cosh}[a]\operatorname{Sinh}[bx]}{b}$$

**Problem 147: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Coth}[c+bx]^2 \operatorname{Sinh}[a+bx] dx$$

Optimal (type 3, 46 leaves, 6 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+bx]] \operatorname{Cosh}[a-c]}{b} + \frac{\operatorname{Cosh}[a+bx]}{b} - \frac{\operatorname{Csch}[c+bx] \operatorname{Sinh}[a-c]}{b}$$

Result (type 3, 110 leaves):

$$-\frac{2 \operatorname{Im} \operatorname{ArcTan} \left[ \frac{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c]) \left( \operatorname{Cosh}[c] \operatorname{Cosh} \left[ \frac{bx}{2} \right] + \operatorname{Sinh}[c] \operatorname{Sinh} \left[ \frac{bx}{2} \right] \right)}{i \operatorname{Cosh}[c] \operatorname{Cosh} \left[ \frac{bx}{2} \right] - i \operatorname{Cosh} \left[ \frac{bx}{2} \right] \operatorname{Sinh}[c]} \right] \operatorname{Cosh}[a - c]}{b} + \frac{\operatorname{Cosh}[a] \operatorname{Cosh}[bx]}{b} - \frac{\operatorname{Csch}[c + bx] \operatorname{Sinh}[a - c]}{b} + \frac{\operatorname{Sinh}[a] \operatorname{Sinh}[bx]}{b}$$

Problem 150: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + bx]^2 \operatorname{Sinh}[a + bx] \, dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{\operatorname{Cosh}[a - c] \operatorname{Sech}[c + bx]}{b} + \frac{\operatorname{ArcTan}[\operatorname{Sinh}[c + bx]] \operatorname{Sinh}[a - c]}{b}$$

Result (type 3, 83 leaves):

$$-\frac{\operatorname{Cosh}[a - c] \operatorname{Sech}[c + bx]}{b} + \frac{2 \operatorname{ArcTan} \left[ \frac{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c]) \left( \operatorname{Cosh} \left[ \frac{bx}{2} \right] \operatorname{Sinh}[c] + \operatorname{Cosh}[c] \operatorname{Sinh} \left[ \frac{bx}{2} \right] \right)}{\operatorname{Cosh}[c] \operatorname{Cosh} \left[ \frac{bx}{2} \right] - \operatorname{Cosh} \left[ \frac{bx}{2} \right] \operatorname{Sinh}[c]} \right] \operatorname{Sinh}[a - c]}{b}$$

Problem 153: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + bx]^2 \operatorname{Sinh}[a + bx] \, dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c + bx]] \operatorname{Cosh}[a - c]}{b} - \frac{\operatorname{Csch}[c + bx] \operatorname{Sinh}[a - c]}{b}$$

Result (type 3, 90 leaves):

$$-\frac{2 \operatorname{Im} \operatorname{ArcTan} \left[ \frac{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c]) \left( \operatorname{Cosh}[c] \operatorname{Cosh} \left[ \frac{bx}{2} \right] + \operatorname{Sinh}[c] \operatorname{Sinh} \left[ \frac{bx}{2} \right] \right)}{i \operatorname{Cosh}[c] \operatorname{Cosh} \left[ \frac{bx}{2} \right] - i \operatorname{Cosh} \left[ \frac{bx}{2} \right] \operatorname{Sinh}[c]} \right] \operatorname{Cosh}[a - c]}{b} - \frac{\operatorname{Csch}[c + bx] \operatorname{Sinh}[a - c]}{b}$$

Problem 155: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cosh}[a + bx] \operatorname{Tanh}[c + bx] \, dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$\frac{\text{Cosh}[a + b x]}{b} - \frac{\text{ArcTan}[\text{Sinh}[c + b x]] \text{Sinh}[a - c]}{b}$$

Result (type 3, 86 leaves):

$$\frac{\text{Cosh}[a] \text{Cosh}[b x]}{b} - \frac{2 \text{ArcTan}\left[\frac{(\text{Cosh}[c] - \text{Sinh}[c]) \left(\text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c] + \text{Cosh}[c] \text{Sinh}\left[\frac{b x}{2}\right]\right)}{\text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] - \text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c]}\right] \text{Sinh}[a - c]}{b} + \frac{\text{Sinh}[a] \text{Sinh}[b x]}{b}$$

Problem 156: Result more than twice size of optimal antiderivative.

$$\int \text{Cosh}[a + b x] \text{Tanh}[c + b x]^2 dx$$

Optimal (type 3, 45 leaves, 6 steps):

$$-\frac{\text{ArcTan}[\text{Sinh}[c + b x]] \text{Cosh}[a - c]}{b} + \frac{\text{Sech}[c + b x] \text{Sinh}[a - c]}{b} + \frac{\text{Sinh}[a + b x]}{b}$$

Result (type 3, 102 leaves):

$$-\frac{2 \text{ArcTan}\left[\frac{(\text{Cosh}[c] - \text{Sinh}[c]) \left(\text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c] + \text{Cosh}[c] \text{Sinh}\left[\frac{b x}{2}\right]\right)}{\text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] - \text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c]}\right] \text{Cosh}[a - c]}{b} + \frac{\text{Cosh}[b x] \text{Sinh}[a]}{b} + \frac{\text{Sech}[c + b x] \text{Sinh}[a - c]}{b} + \frac{\text{Cosh}[a] \text{Sinh}[b x]}{b}$$

Problem 158: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Cosh}[a + b x] \text{Coth}[c + b x] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$-\frac{\text{ArcTanh}[\text{Cosh}[c + b x]] \text{Cosh}[a - c]}{b} + \frac{\text{Cosh}[a + b x]}{b}$$

Result (type 3, 93 leaves):

$$-\frac{2 i \text{ArcTan}\left[\frac{(\text{Cosh}[c] - \text{Sinh}[c]) \left(\text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] + \text{Sinh}[c] \text{Sinh}\left[\frac{b x}{2}\right]\right)}{i \text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] - i \text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c]}\right] \text{Cosh}[a - c]}{b} + \frac{\text{Cosh}[a] \text{Cosh}[b x]}{b} + \frac{\text{Sinh}[a] \text{Sinh}[b x]}{b}$$

Problem 159: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Cosh}[a + b x] \text{Coth}[c + b x]^2 dx$$



Optimal (type 3, 46 leaves, 6 steps):

$$-\frac{\text{Cosh}[a - c] \text{Csch}[c + b x]}{b} - \frac{\text{ArcTanh}[\text{Cosh}[c + b x]] \text{Sinh}[a - c]}{b} + \frac{\text{Sinh}[a + b x]}{b}$$

Result (type 3, 110 leaves):

$$-\frac{\text{Cosh}[a - c] \text{Csch}[c + b x]}{b} + \frac{\text{Cosh}[b x] \text{Sinh}[a]}{b} - \frac{2 i \text{ArcTan}\left[\frac{(\text{Cosh}[c] - \text{Sinh}[c]) (\text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] + \text{Sinh}[c] \text{Sinh}\left[\frac{b x}{2}\right])}{i \text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] - i \text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c]}\right] \text{Sinh}[a - c]}{b} + \frac{\text{Cosh}[a] \text{Sinh}[b x]}{b}$$

**Problem 162: Result more than twice size of optimal antiderivative.**

$$\int \text{Cosh}[a + b x] \text{Sech}[c + b x]^2 dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$\frac{\text{ArcTan}[\text{Sinh}[c + b x]] \text{Cosh}[a - c]}{b} - \frac{\text{Sech}[c + b x] \text{Sinh}[a - c]}{b}$$

Result (type 3, 83 leaves):

$$\frac{2 \text{ArcTan}\left[\frac{(\text{Cosh}[c] - \text{Sinh}[c]) (\text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c] + \text{Cosh}[c] \text{Sinh}\left[\frac{b x}{2}\right])}{\text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] - \text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c]}\right] \text{Cosh}[a - c]}{b} - \frac{\text{Sech}[c + b x] \text{Sinh}[a - c]}{b}$$

**Problem 165: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cosh}[a + b x] \text{Csch}[c + b x]^2 dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{\text{Cosh}[a - c] \text{Csch}[c + b x]}{b} - \frac{\text{ArcTanh}[\text{Cosh}[c + b x]] \text{Sinh}[a - c]}{b}$$

Result (type 3, 90 leaves):

$$-\frac{\text{Cosh}[a - c] \text{Csch}[c + b x]}{b} - \frac{2 i \text{ArcTan}\left[\frac{(\text{Cosh}[c] - \text{Sinh}[c]) (\text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] + \text{Sinh}[c] \text{Sinh}\left[\frac{b x}{2}\right])}{i \text{Cosh}[c] \text{Cosh}\left[\frac{b x}{2}\right] - i \text{Cosh}\left[\frac{b x}{2}\right] \text{Sinh}[c]}\right] \text{Sinh}[a - c]}{b}$$

### Problem 188: Result more than twice size of optimal antiderivative.

$$\int \text{Sinh}[a + b x] \text{Tanh}[c + d x] dx$$

Optimal (type 5, 121 leaves, 6 steps):

$$\frac{e^{-a-bx}}{2b} + \frac{e^{a+bx}}{2b} - \frac{e^{-a-bx} \text{Hypergeometric2F1}\left[1, -\frac{b}{2d}, 1 - \frac{b}{2d}, -e^{2(c+dx)}\right]}{b} - \frac{e^{a+bx} \text{Hypergeometric2F1}\left[1, \frac{b}{2d}, 1 + \frac{b}{2d}, -e^{2(c+dx)}\right]}{b}$$

Result (type 5, 278 leaves):

$$\begin{aligned} & \frac{1}{4(b^3 - 4bd^2)} e^{-a-c-bx} \left( -b(b+2d) e^{2(c+dx)} (-1 + e^{2a}) \text{Hypergeometric2F1}\left[1, 1 - \frac{b}{2d}, 2 - \frac{b}{2d}, -e^{2(c+dx)}\right] \text{Sech}[c] + \right. \\ & (b-2d) \left( 2b e^{2(a+c+(b+d)x)} \text{Hypergeometric2F1}\left[1, 1 + \frac{b}{2d}, 2 + \frac{b}{2d}, -e^{2(c+dx)}\right] \text{Sech}[c] - \right. \\ & (b+2d) \left( -\text{Sech}[c] - e^{2a} \text{Sech}[c] + (1 + e^{2a} + 2e^{2c}) \text{Hypergeometric2F1}\left[1, -\frac{b}{2d}, 1 - \frac{b}{2d}, -e^{2(c+dx)}\right] \text{Sech}[c] + \right. \\ & \left. \left. \left. 2e^{2(a+c+bx)} \text{Hypergeometric2F1}\left[1, \frac{b}{2d}, 1 + \frac{b}{2d}, -e^{2(c+dx)}\right] \text{Sech}[c] - 4e^{a+c+bx} \text{Cosh}[a+bx] \text{Tanh}[c] \right) \right) \right) \end{aligned}$$

### Problem 189: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[c + d x] \text{Sinh}[a + b x] dx$$

Optimal (type 5, 117 leaves, 6 steps):

$$\frac{e^{-a-bx}}{2b} + \frac{e^{a+bx}}{2b} - \frac{e^{-a-bx} \text{Hypergeometric2F1}\left[1, -\frac{b}{2d}, 1 - \frac{b}{2d}, e^{2(c+dx)}\right]}{b} - \frac{e^{a+bx} \text{Hypergeometric2F1}\left[1, \frac{b}{2d}, 1 + \frac{b}{2d}, e^{2(c+dx)}\right]}{b}$$

Result (type 5, 240 leaves):

$$\begin{aligned} & \frac{\text{Cosh}[a] \text{Cosh}[bx] \text{Coth}[c]}{b} + \frac{1}{b(b-2d)(-1+e^{2c})} \\ & e^{-a+2c-bx} \left( b e^{2dx} \text{Hypergeometric2F1}\left[1, 1 - \frac{b}{2d}, 2 - \frac{b}{2d}, e^{2(c+dx)}\right] - (b-2d) \text{Hypergeometric2F1}\left[1, -\frac{b}{2d}, 1 - \frac{b}{2d}, e^{2(c+dx)}\right] \right) - \\ & \frac{e^{a+2c} \left( -\frac{e^{(b+2d)x} \text{Hypergeometric2F1}\left[1, 1 + \frac{b}{2d}, 2 + \frac{b}{2d}, e^{2(c+dx)}\right]}{b+2d} + \frac{e^{bx} \text{Hypergeometric2F1}\left[1, \frac{b}{2d}, 1 + \frac{b}{2d}, e^{2(c+dx)}\right]}{b} \right)}{-1+e^{2c}} + \frac{\text{Coth}[c] \text{Sinh}[a] \text{Sinh}[bx]}{b} \end{aligned}$$

**Problem 200: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sinh}[x] \text{Tanh}[2x] dx$$

Optimal (type 3, 19 leaves, 4 steps):

$$-\frac{\text{ArcTan}[\sqrt{2} \text{Sinh}[x]]}{\sqrt{2}} + \text{Sinh}[x]$$

Result (type 3, 167 leaves):

$$\frac{1}{4\sqrt{2}} \left( -2 \text{ArcTan} \left[ \frac{\text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \text{Cosh}\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \text{Sinh}\left[\frac{x}{2}\right]} \right] - 2 \text{ArcTan} \left[ \frac{\text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \text{Cosh}\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \text{Sinh}\left[\frac{x}{2}\right]} \right] - \right. \\ \left. 2 \text{ArcTan}[\sqrt{2} \text{Sinh}[x]] + i \text{Log}[\sqrt{2} - 2 \text{Cosh}[x]] + i \text{Log}[\sqrt{2} + 2 \text{Cosh}[x]] - i \text{Log}[\text{Cosh}[2x]] + 4\sqrt{2} \text{Sinh}[x] \right)$$

**Problem 202: Result is not expressed in closed-form.**

$$\int \text{Sinh}[x] \text{Tanh}[4x] dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$-\frac{1}{4} \sqrt{2 - \sqrt{2}} \text{ArcTan} \left[ \frac{2 \text{Sinh}[x]}{\sqrt{2 - \sqrt{2}}} \right] - \frac{1}{4} \sqrt{2 + \sqrt{2}} \text{ArcTan} \left[ \frac{2 \text{Sinh}[x]}{\sqrt{2 + \sqrt{2}}} \right] + \text{Sinh}[x]$$

Result (type 7, 111 leaves):

$$-\frac{1}{16} \text{RootSum} \left[ 1 + \#1^8 \&, \frac{1}{\#1^7} \right. \\ \left. \left( x + 2 \text{Log} \left[ -\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1 \right] + x \#1^6 + 2 \text{Log} \left[ -\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1 \right] \#1^6 \right) \& \right] + \text{Sinh}[x]$$

**Problem 203: Result is not expressed in closed-form.**

$$\int \text{Sinh}[x] \text{Tanh}[5x] dx$$

Optimal (type 3, 87 leaves, 9 steps):

$$-\frac{1}{5} \operatorname{ArcTan}[\operatorname{Sinh}[x]] - \frac{1}{5} \sqrt{\frac{1}{2}(3+\sqrt{5})} \operatorname{ArcTan}\left[2 \sqrt{\frac{2}{3+\sqrt{5}}} \operatorname{Sinh}[x]\right] - \frac{1}{5} \sqrt{\frac{1}{2}(3-\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{2(3+\sqrt{5})} \operatorname{Sinh}[x]\right] + \operatorname{Sinh}[x]$$

Result (type 7, 262 leaves):

$$-\frac{2}{5} \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right] - \frac{1}{20} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 - \#1^6 + \#1^8 \&, \right. \\ \left. \left(3x + 6 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{x}{2}\right] - \operatorname{Sinh}\left[\frac{x}{2}\right] + \operatorname{Cosh}\left[\frac{x}{2}\right] \#1 - \operatorname{Sinh}\left[\frac{x}{2}\right] \#1\right] - x \#1^2 - 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{x}{2}\right] - \operatorname{Sinh}\left[\frac{x}{2}\right] + \operatorname{Cosh}\left[\frac{x}{2}\right] \#1 - \operatorname{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^2 - \right. \right. \\ \left. \left. x \#1^4 - 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{x}{2}\right] - \operatorname{Sinh}\left[\frac{x}{2}\right] + \operatorname{Cosh}\left[\frac{x}{2}\right] \#1 - \operatorname{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^4 + 3x \#1^6 + \right. \right. \\ \left. \left. 6 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{x}{2}\right] - \operatorname{Sinh}\left[\frac{x}{2}\right] + \operatorname{Cosh}\left[\frac{x}{2}\right] \#1 - \operatorname{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^6\right) / \left(-\#1 + 2 \#1^3 - 3 \#1^5 + 4 \#1^7\right) \& \right] + \operatorname{Sinh}[x]$$

Problem 204: Result is not expressed in closed-form.

$$\int \operatorname{Sinh}[x] \operatorname{Tanh}[6x] dx$$

Optimal (type 3, 87 leaves, 10 steps):

$$-\frac{\operatorname{ArcTan}\left[\sqrt{2} \operatorname{Sinh}[x]\right]}{3\sqrt{2}} - \frac{1}{6} \sqrt{2-\sqrt{3}} \operatorname{ArcTan}\left[\frac{2 \operatorname{Sinh}[x]}{\sqrt{2-\sqrt{3}}}\right] - \frac{1}{6} \sqrt{2+\sqrt{3}} \operatorname{ArcTan}\left[\frac{2 \operatorname{Sinh}[x]}{\sqrt{2+\sqrt{3}}}\right] + \operatorname{Sinh}[x]$$

Result (type 7, 397 leaves):

$$-\frac{1}{24\sqrt{2}} \left( 4 \operatorname{ArcTan}\left[\frac{\operatorname{Cosh}\left[\frac{x}{2}\right] + \operatorname{Sinh}\left[\frac{x}{2}\right]}{(1+\sqrt{2}) \operatorname{Cosh}\left[\frac{x}{2}\right] - (-1+\sqrt{2}) \operatorname{Sinh}\left[\frac{x}{2}\right]}\right] + 4 \operatorname{ArcTan}\left[\frac{\operatorname{Cosh}\left[\frac{x}{2}\right] + \operatorname{Sinh}\left[\frac{x}{2}\right]}{(-1+\sqrt{2}) \operatorname{Cosh}\left[\frac{x}{2}\right] - (1+\sqrt{2}) \operatorname{Sinh}\left[\frac{x}{2}\right]}\right] + \right. \\ \left. 4 \operatorname{ArcTan}\left[\sqrt{2} \operatorname{Sinh}[x]\right] - 2 \operatorname{Log}\left[\sqrt{2} - 2 \operatorname{Cosh}[x]\right] - 2 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Cosh}[x]\right] + 2 \operatorname{Log}\left[\operatorname{Cosh}[2x]\right] + \right. \\ \left. \sqrt{2} \operatorname{RootSum}\left[1 - \#1^4 + \#1^8 \&, \frac{1}{-\#1^3 + 2 \#1^7} \left(2x + 4 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{x}{2}\right] - \operatorname{Sinh}\left[\frac{x}{2}\right] + \operatorname{Cosh}\left[\frac{x}{2}\right] \#1 - \operatorname{Sinh}\left[\frac{x}{2}\right] \#1\right] - x \#1^2 - \right. \right. \right. \\ \left. \left. 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{x}{2}\right] - \operatorname{Sinh}\left[\frac{x}{2}\right] + \operatorname{Cosh}\left[\frac{x}{2}\right] \#1 - \operatorname{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^2 - x \#1^4 - 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{x}{2}\right] - \operatorname{Sinh}\left[\frac{x}{2}\right] + \operatorname{Cosh}\left[\frac{x}{2}\right] \#1 - \operatorname{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^4 + \right. \right. \\ \left. \left. 2x \#1^6 + 4 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{x}{2}\right] - \operatorname{Sinh}\left[\frac{x}{2}\right] + \operatorname{Cosh}\left[\frac{x}{2}\right] \#1 - \operatorname{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^6\right) \& \right] - 24\sqrt{2} \operatorname{Sinh}[x] \left. \right)$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sinh}[x] \operatorname{Tanh}[nx] dx$$

Optimal (type 5, 81 leaves, 6 steps):

$$\frac{e^{-x}}{2} + \frac{e^x}{2} - e^{-x} \operatorname{Hypergeometric2F1}\left[1, -\frac{1}{2n}, 1 - \frac{1}{2n}, -e^{2nx}\right] - e^x \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2n}, \frac{1}{2} \left(2 + \frac{1}{n}\right), -e^{2nx}\right]$$

Result (type 5, 164 leaves):

$$\frac{1}{2} e^{-2x} \left( -\frac{e^{x+2nx} \operatorname{Hypergeometric2F1}\left[1, 1 - \frac{1}{2n}, 2 - \frac{1}{2n}, -e^{2nx}\right]}{-1 + 2n} + \frac{e^{(3+2n)x} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{2n}, 2 + \frac{1}{2n}, -e^{2nx}\right]}{1 + 2n} - e^x \left( \operatorname{Hypergeometric2F1}\left[1, -\frac{1}{2n}, 1 - \frac{1}{2n}, -e^{2nx}\right] + e^{2x} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2n}, 1 + \frac{1}{2n}, -e^{2nx}\right] \right) \right)$$

**Problem 208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Coth}[4x] \operatorname{Sinh}[x] dx$$

Optimal (type 3, 28 leaves, 6 steps):

$$-\frac{1}{4} \operatorname{ArcTan}[\operatorname{Sinh}[x]] - \frac{\operatorname{ArcTan}[\sqrt{2} \operatorname{Sinh}[x]]}{2\sqrt{2}} + \operatorname{Sinh}[x]$$

Result (type 3, 181 leaves):

$$-\frac{1}{8\sqrt{2}} \left( 2 \operatorname{ArcTan}\left[ \frac{\operatorname{Cosh}\left[\frac{x}{2}\right] + \operatorname{Sinh}\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \operatorname{Cosh}\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \operatorname{Sinh}\left[\frac{x}{2}\right]} \right] + 2 \operatorname{ArcTan}\left[ \frac{\operatorname{Cosh}\left[\frac{x}{2}\right] + \operatorname{Sinh}\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \operatorname{Cosh}\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \operatorname{Sinh}\left[\frac{x}{2}\right]} \right] + 2 \operatorname{ArcTan}[\sqrt{2} \operatorname{Sinh}[x]] + 4\sqrt{2} \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right] - i \operatorname{Log}[\sqrt{2} - 2 \operatorname{Cosh}[x]] - i \operatorname{Log}[\sqrt{2} + 2 \operatorname{Cosh}[x]] + i \operatorname{Log}[\operatorname{Cosh}[2x]] - 8\sqrt{2} \operatorname{Sinh}[x] \right)$$

**Problem 209: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Coth}[5x] \operatorname{Sinh}[x] dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$-\frac{1}{5} \sqrt{\frac{1}{2} (5 + \sqrt{5})} \operatorname{ArcTan}\left[2 \sqrt{\frac{2}{5 + \sqrt{5}}} \operatorname{Sinh}[x]\right] - \frac{1}{5} \sqrt{\frac{1}{2} (5 - \sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{5 + \sqrt{5}}} \operatorname{Sinh}[x]\right] + \operatorname{Sinh}[x]$$

Result (type 3, 198 leaves):

$$\frac{1}{20\sqrt{5}} \left( -(-5 + \sqrt{5}) \sqrt{2(5 + \sqrt{5})} \operatorname{ArcTan} \left[ \frac{(-3 + \sqrt{5}) \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{10 - 2\sqrt{5}}} \right] + (-5 + \sqrt{5}) \sqrt{2(5 + \sqrt{5})} \operatorname{ArcTan} \left[ \frac{(5 + \sqrt{5}) \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{10 - 2\sqrt{5}}} \right] + \right. \\ \left. \sqrt{10 - 2\sqrt{5}} (5 + \sqrt{5}) \left( \operatorname{ArcTan} \left[ \frac{(-5 + \sqrt{5}) \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{2(5 + \sqrt{5})}} \right] - \operatorname{ArcTan} \left[ \frac{(3 + \sqrt{5}) \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{2(5 + \sqrt{5})}} \right] \right) + 20\sqrt{5} \operatorname{Sinh}[x] \right)$$

**Problem 211:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[2x] \operatorname{Sinh}[x] dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sqrt{2} \operatorname{Cosh}[x]]}{\sqrt{2}}$$

Result (type 3, 155 leaves):

$$\frac{1}{4\sqrt{2}} \left( -2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(1 + \sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (-1 + \sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] + \right. \\ \left. 2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(-1 + \sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (1 + \sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] - 4 \operatorname{ArcTanh}[\sqrt{2} - i \operatorname{Tanh} \left[ \frac{x}{2} \right]] + \operatorname{Log}[\sqrt{2} - 2 \operatorname{Cosh}[x]] - \operatorname{Log}[\sqrt{2} + 2 \operatorname{Cosh}[x]] \right)$$

**Problem 213:** Result is not expressed in closed-form.

$$\int \operatorname{Sech}[4x] \operatorname{Sinh}[x] dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh} \left[ \frac{2 \operatorname{Cosh}[x]}{\sqrt{2 - \sqrt{2}}} \right]}{2\sqrt{2(2 - \sqrt{2})}} - \frac{\operatorname{ArcTanh} \left[ \frac{2 \operatorname{Cosh}[x]}{\sqrt{2 + \sqrt{2}}} \right]}{2\sqrt{2(2 + \sqrt{2})}}$$

Result (type 7, 110 leaves):

$$\frac{1}{16} \text{RootSum}\left[1 + \#1^8 \&, \frac{1}{\#1^5} \left(-x - 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] + x \#1^2 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^2\right) \&\right]$$

**Problem 215: Result is not expressed in closed-form.**

$$\int \text{Sech}[6x] \text{Sinh}[x] dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\sqrt{2} \text{Cosh}[x]\right]}{3\sqrt{2}} - \frac{\text{ArcTanh}\left[\frac{2\text{Cosh}[x]}{\sqrt{2-\sqrt{3}}}\right]}{6\sqrt{2-\sqrt{3}}} - \frac{\text{ArcTanh}\left[\frac{2\text{Cosh}[x]}{\sqrt{2+\sqrt{3}}}\right]}{6\sqrt{2+\sqrt{3}}}$$

Result (type 7, 385 leaves):

$$\frac{1}{24\sqrt{2}} \left( 4 \text{ArcTan}\left[\frac{\text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]}{(1+\sqrt{2})\text{Cosh}\left[\frac{x}{2}\right] - (-1+\sqrt{2})\text{Sinh}\left[\frac{x}{2}\right]}\right] - 4 \text{ArcTan}\left[\frac{\text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]}{(-1+\sqrt{2})\text{Cosh}\left[\frac{x}{2}\right] - (1+\sqrt{2})\text{Sinh}\left[\frac{x}{2}\right]}\right] + 8 \text{ArcTanh}\left[\sqrt{2} - \text{Tanh}\left[\frac{x}{2}\right]\right] - 2 \text{Log}\left[\sqrt{2} - 2 \text{Cosh}[x]\right] + 2 \text{Log}\left[\sqrt{2} + 2 \text{Cosh}[x]\right] + \sqrt{2} \text{RootSum}\left[1 - \#1^4 + \#1^8 \&, \frac{1}{-\#1^3 + 2\#1^7} \left(-x - 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] + x \#1^2 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^2 - x \#1^4 - 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^4 + x \#1^6 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^6\right) \&\right]$$

**Problem 218: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Csch}[4x] \text{Sinh}[x] dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4} \text{ArcTan}[\text{Sinh}[x]] + \frac{\text{ArcTan}\left[\sqrt{2} \text{Sinh}[x]\right]}{2\sqrt{2}}$$

Result (type 3, 172 leaves):

$$-\frac{1}{8\sqrt{2}}i \left( 2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(1+\sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (-1+\sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] + 2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(-1+\sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (1+\sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] + \right. \\ \left. 2i \operatorname{ArcTan} [\sqrt{2} \operatorname{Sinh} [x]] - 4i \sqrt{2} \operatorname{ArcTan} [\operatorname{Tanh} \left[ \frac{x}{2} \right]] + \operatorname{Log} [\sqrt{2} - 2 \operatorname{Cosh} [x]] + \operatorname{Log} [\sqrt{2} + 2 \operatorname{Cosh} [x]] - \operatorname{Log} [\operatorname{Cosh} [2x]] \right)$$

**Problem 229: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cosh} [x] \operatorname{Tanh} [2x] dx$$

Optimal (type 3, 19 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh} [\sqrt{2} \operatorname{Cosh} [x]]}{\sqrt{2}} + \operatorname{Cosh} [x]$$

Result (type 3, 164 leaves):

$$\frac{1}{4\sqrt{2}} \left( -2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(1+\sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (-1+\sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] + 2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(-1+\sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (1+\sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] - \right. \\ \left. 4 \operatorname{ArcTanh} [\sqrt{2} - i \operatorname{Tanh} \left[ \frac{x}{2} \right]] + 4\sqrt{2} \operatorname{Cosh} [x] + \operatorname{Log} [\sqrt{2} - 2 \operatorname{Cosh} [x]] - \operatorname{Log} [\sqrt{2} + 2 \operatorname{Cosh} [x]] \right)$$

**Problem 230: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cosh} [x] \operatorname{Tanh} [3x] dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh} \left[ \frac{2 \operatorname{Cosh} [x]}{\sqrt{3}} \right]}{\sqrt{3}} + \operatorname{Cosh} [x]$$

Result (type 3, 55 leaves):

$$-\frac{\operatorname{ArcTanh} \left[ \frac{2-i \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right]}{\sqrt{3}} - \frac{\operatorname{ArcTanh} \left[ \frac{2+i \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right]}{\sqrt{3}} + \operatorname{Cosh} [x]$$



**Problem 231: Result is not expressed in closed-form.**

$$\int \text{Cosh}[x] \text{Tanh}[4x] dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$-\frac{1}{4} \sqrt{2-\sqrt{2}} \text{ArcTanh}\left[\frac{2 \text{Cosh}[x]}{\sqrt{2-\sqrt{2}}}\right] - \frac{1}{4} \sqrt{2+\sqrt{2}} \text{ArcTanh}\left[\frac{2 \text{Cosh}[x]}{\sqrt{2+\sqrt{2}}}\right] + \text{Cosh}[x]$$

Result (type 7, 113 leaves):

$$\text{Cosh}[x] + \frac{1}{16} \text{RootSum}\left[1 + \#1^8 \&, \frac{1}{\#1^7} \left(-x - 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] + x \#1^6 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^6\right) \&\right]$$

**Problem 232: Result is not expressed in closed-form.**

$$\int \text{Cosh}[x] \text{Tanh}[5x] dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$-\frac{1}{5} \sqrt{\frac{1}{2}(5+\sqrt{5})} \text{ArcTanh}\left[2 \sqrt{\frac{2}{5+\sqrt{5}}} \text{Cosh}[x]\right] - \frac{1}{5} \sqrt{\frac{1}{2}(5-\sqrt{5})} \text{ArcTanh}\left[\sqrt{\frac{2}{5}} (5+\sqrt{5}) \text{Cosh}[x]\right] + \text{Cosh}[x]$$

Result (type 7, 249 leaves):

$$\text{Cosh}[x] + \frac{1}{4} \text{RootSum}\left[1 - \#1^2 + \#1^4 - \#1^6 + \#1^8 \&, \left(-x - 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] + x \#1^2 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^2 - x \#1^4 - 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^4 + x \#1^6 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^6\right) / (-\#1 + 2 \#1^3 - 3 \#1^5 + 4 \#1^7) \&\right]$$

**Problem 233: Result is not expressed in closed-form.**

$$\int \text{Cosh}[x] \text{Tanh}[6x] dx$$

Optimal (type 3, 87 leaves, 10 steps):

$$-\frac{\text{ArcTanh}[\sqrt{2} \text{Cosh}[x]]}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}} \text{ArcTanh}\left[\frac{2\text{Cosh}[x]}{\sqrt{2-\sqrt{3}}}\right] - \frac{1}{6}\sqrt{2+\sqrt{3}} \text{ArcTanh}\left[\frac{2\text{Cosh}[x]}{\sqrt{2+\sqrt{3}}}\right] + \text{Cosh}[x]$$

Result (type 7, 395 leaves):

$$\frac{1}{24\sqrt{2}} \left( -4i \text{ArcTan}\left[\frac{\text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]}{(1+\sqrt{2})\text{Cosh}\left[\frac{x}{2}\right] - (-1+\sqrt{2})\text{Sinh}\left[\frac{x}{2}\right]}\right] + 4i \text{ArcTan}\left[\frac{\text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]}{(-1+\sqrt{2})\text{Cosh}\left[\frac{x}{2}\right] - (1+\sqrt{2})\text{Sinh}\left[\frac{x}{2}\right]}\right] - \right. \\ \left. 8 \text{ArcTanh}\left[\sqrt{2} - i \text{Tanh}\left[\frac{x}{2}\right]\right] + 24\sqrt{2} \text{Cosh}[x] + 2 \text{Log}\left[\sqrt{2} - 2\text{Cosh}[x]\right] - 2 \text{Log}\left[\sqrt{2} + 2\text{Cosh}[x]\right] + \sqrt{2} \text{RootSum}\left[1 - \#1^4 + \#1^8 \&, \frac{1}{-\#1^3 + 2\#1^7}\right] \right. \\ \left. \left( -2x - 4 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] - x \#1^2 - 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^2 + x \#1^4 + \right. \right. \\ \left. \left. 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^4 + 2x \#1^6 + 4 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^6 \right) \& \right)$$

**Problem 234: Result more than twice size of optimal antiderivative.**

$$\int \text{Cosh}[x] \text{Coth}[2x] dx$$

Optimal (type 3, 10 leaves, 4 steps):

$$-\frac{1}{2} \text{ArcTanh}[\text{Cosh}[x]] + \text{Cosh}[x]$$

Result (type 3, 25 leaves):

$$\text{Cosh}[x] - \frac{1}{2} \text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] + \frac{1}{2} \text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]$$

**Problem 236: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cosh}[x] \text{Coth}[4x] dx$$

Optimal (type 3, 28 leaves, 6 steps):

$$-\frac{1}{4} \text{ArcTanh}[\text{Cosh}[x]] - \frac{\text{ArcTanh}[\sqrt{2} \text{Cosh}[x]]}{2\sqrt{2}} + \text{Cosh}[x]$$

Result (type 3, 192 leaves):

$$\frac{1}{8\sqrt{2}} \left( -2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(1 + \sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (-1 + \sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] + 2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(-1 + \sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (1 + \sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] - \right. \\ \left. 4 \operatorname{ArcTanh} \left[ \sqrt{2} - i \operatorname{Tanh} \left[ \frac{x}{2} \right] \right] + 8\sqrt{2} \operatorname{Cosh} [x] - 2\sqrt{2} \operatorname{Log} \left[ \operatorname{Cosh} \left[ \frac{x}{2} \right] \right] + \operatorname{Log} \left[ \sqrt{2} - 2 \operatorname{Cosh} [x] \right] - \operatorname{Log} \left[ \sqrt{2} + 2 \operatorname{Cosh} [x] \right] + 2\sqrt{2} \operatorname{Log} \left[ \operatorname{Sinh} \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 238: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cosh} [x] \operatorname{Coth} [6x] dx$$

Optimal (type 3, 38 leaves, 7 steps):

$$-\frac{1}{6} \operatorname{ArcTanh} [\operatorname{Cosh} [x]] - \frac{1}{6} \operatorname{ArcTanh} [2 \operatorname{Cosh} [x]] - \frac{\operatorname{ArcTanh} \left[ \frac{2 \operatorname{Cosh} [x]}{\sqrt{3}} \right]}{2\sqrt{3}} + \operatorname{Cosh} [x]$$

Result (type 3, 95 leaves):

$$\frac{1}{12} \left( -2\sqrt{3} \operatorname{ArcTanh} \left[ \frac{2 - i \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] - 2\sqrt{3} \operatorname{ArcTanh} \left[ \frac{2 + i \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] + \right. \\ \left. 12 \operatorname{Cosh} [x] - 2 \operatorname{Log} \left[ \operatorname{Cosh} \left[ \frac{x}{2} \right] \right] + \operatorname{Log} [1 - 2 \operatorname{Cosh} [x]] - \operatorname{Log} [1 + 2 \operatorname{Cosh} [x]] + 2 \operatorname{Log} \left[ \operatorname{Sinh} \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 239: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cosh} [x] \operatorname{Coth} [nx] dx$$

Optimal (type 5, 76 leaves, 6 steps):

$$-\frac{e^{-x}}{2} + \frac{e^x}{2} + e^{-x} \operatorname{Hypergeometric2F1} \left[ 1, -\frac{1}{2n}, 1 - \frac{1}{2n}, e^{2nx} \right] - e^x \operatorname{Hypergeometric2F1} \left[ 1, \frac{1}{2n}, 1 + \frac{1}{2n}, e^{2nx} \right]$$

Result (type 5, 156 leaves):

$$\frac{1}{2} e^{-2x} \left( -\frac{e^{x+2nx} \operatorname{Hypergeometric2F1} \left[ 1, 1 - \frac{1}{2n}, 2 - \frac{1}{2n}, e^{2nx} \right]}{-1 + 2n} - \frac{e^{(3+2n)x} \operatorname{Hypergeometric2F1} \left[ 1, 1 + \frac{1}{2n}, 2 + \frac{1}{2n}, e^{2nx} \right]}{1 + 2n} + \right. \\ \left. e^x \operatorname{Hypergeometric2F1} \left[ 1, -\frac{1}{2n}, 1 - \frac{1}{2n}, e^{2nx} \right] - e^{3x} \operatorname{Hypergeometric2F1} \left[ 1, \frac{1}{2n}, 1 + \frac{1}{2n}, e^{2nx} \right] \right)$$

**Problem 240: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cosh}[x] \text{Sech}[2x] dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\text{ArcTan}[\sqrt{2} \text{Sinh}[x]]}{\sqrt{2}}$$

Result (type 3, 156 leaves):

$$-\frac{1}{4\sqrt{2}} i \left( 2 i \text{ArcTan} \left[ \frac{\text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \text{Cosh}\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \text{Sinh}\left[\frac{x}{2}\right]} \right] + 2 i \text{ArcTan} \left[ \frac{\text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \text{Cosh}\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \text{Sinh}\left[\frac{x}{2}\right]} \right] + \right. \\ \left. 2 i \text{ArcTan}[\sqrt{2} \text{Sinh}[x]] + \text{Log}[\sqrt{2} - 2 \text{Cosh}[x]] + \text{Log}[\sqrt{2} + 2 \text{Cosh}[x]] - \text{Log}[\text{Cosh}[2x]] \right)$$

**Problem 242: Result is not expressed in closed-form.**

$$\int \text{Cosh}[x] \text{Sech}[4x] dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{2\text{Sinh}[x]}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2(2-\sqrt{2})}} - \frac{\text{ArcTan}\left[\frac{2\text{Sinh}[x]}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2(2+\sqrt{2})}}$$

Result (type 7, 108 leaves):

$$\frac{1}{16} \text{RootSum}\left[1 + \#1^8 \&, \right. \\ \left. \frac{1}{\#1^5} \left( x + 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] + x \#1^2 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^2 \right) \& \right]$$

**Problem 244: Result is not expressed in closed-form.**

$$\int \text{Cosh}[x] \text{Sech}[6x] dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$-\frac{\text{ArcTan}[\sqrt{2} \sinh[x]]}{3\sqrt{2}} + \frac{\text{ArcTan}\left[\frac{2\sinh[x]}{\sqrt{2-\sqrt{3}}}\right]}{6\sqrt{2-\sqrt{3}}} + \frac{\text{ArcTan}\left[\frac{2\sinh[x]}{\sqrt{2+\sqrt{3}}}\right]}{6\sqrt{2+\sqrt{3}}}$$

Result (type 7, 383 leaves):

$$\frac{1}{24\sqrt{2}} \left( -4 \text{ArcTan}\left[\frac{\cosh\left[\frac{x}{2}\right] + \sinh\left[\frac{x}{2}\right]}{(1+\sqrt{2})\cosh\left[\frac{x}{2}\right] - (-1+\sqrt{2})\sinh\left[\frac{x}{2}\right]}\right] - 4 \text{ArcTan}\left[\frac{\cosh\left[\frac{x}{2}\right] + \sinh\left[\frac{x}{2}\right]}{(-1+\sqrt{2})\cosh\left[\frac{x}{2}\right] - (1+\sqrt{2})\sinh\left[\frac{x}{2}\right]}\right] - 4 \text{ArcTan}[\sqrt{2} \sinh[x]] + \right. \\ \left. 2 \operatorname{Li}\left[\sqrt{2} - 2 \cosh[x]\right] + 2 \operatorname{Li}\left[\sqrt{2} + 2 \cosh[x]\right] - 2 \operatorname{Li}[\cosh[2x]] + \sqrt{2} \operatorname{RootSum}\left[1 - \#1^4 + \#1^8, \frac{1}{-\#1^3 + 2\#1^7}\right] \right. \\ \left. \left( x + 2 \log\left[-\cosh\left[\frac{x}{2}\right] - \sinh\left[\frac{x}{2}\right] + \cosh\left[\frac{x}{2}\right] \#1 - \sinh\left[\frac{x}{2}\right] \#1\right] + x \#1^2 + 2 \log\left[-\cosh\left[\frac{x}{2}\right] - \sinh\left[\frac{x}{2}\right] + \cosh\left[\frac{x}{2}\right] \#1 - \sinh\left[\frac{x}{2}\right] \#1\right] \#1^2 + x \#1^4 + \right. \right. \\ \left. \left. 2 \log\left[-\cosh\left[\frac{x}{2}\right] - \sinh\left[\frac{x}{2}\right] + \cosh\left[\frac{x}{2}\right] \#1 - \sinh\left[\frac{x}{2}\right] \#1\right] \#1^4 + x \#1^6 + 2 \log\left[-\cosh\left[\frac{x}{2}\right] - \sinh\left[\frac{x}{2}\right] + \cosh\left[\frac{x}{2}\right] \#1 - \sinh\left[\frac{x}{2}\right] \#1\right] \#1^6 \right) \& \right)$$

Problem 245: Result more than twice size of optimal antiderivative.

$$\int \cosh[x] \operatorname{csch}[2x] dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\cosh[x]]$$

Result (type 3, 21 leaves):

$$\frac{1}{2} \left( -\log\left[\cosh\left[\frac{x}{2}\right]\right] + \log\left[\sinh\left[\frac{x}{2}\right]\right] \right)$$

Problem 247: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cosh[x] \operatorname{csch}[4x] dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4} \operatorname{ArcTanh}[\cosh[x]] + \frac{\operatorname{ArcTanh}[\sqrt{2} \cosh[x]]}{2\sqrt{2}}$$

Result (type 3, 183 leaves):

$$\frac{1}{8\sqrt{2}} \left( 2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(1 + \sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (-1 + \sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] - 2i \operatorname{ArcTan} \left[ \frac{\operatorname{Cosh} \left[ \frac{x}{2} \right] + \operatorname{Sinh} \left[ \frac{x}{2} \right]}{(-1 + \sqrt{2}) \operatorname{Cosh} \left[ \frac{x}{2} \right] - (1 + \sqrt{2}) \operatorname{Sinh} \left[ \frac{x}{2} \right]} \right] + \right. \\ \left. 4 \operatorname{ArcTanh} \left[ \sqrt{2} - i \operatorname{Tanh} \left[ \frac{x}{2} \right] \right] - 2\sqrt{2} \operatorname{Log} \left[ \operatorname{Cosh} \left[ \frac{x}{2} \right] \right] - \operatorname{Log} \left[ \sqrt{2} - 2 \operatorname{Cosh} [x] \right] + \operatorname{Log} \left[ \sqrt{2} + 2 \operatorname{Cosh} [x] \right] + 2\sqrt{2} \operatorname{Log} \left[ \operatorname{Sinh} \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 249:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{Cosh} [x] \operatorname{Csch} [6x] dx$$

Optimal (type 3, 36 leaves, 7 steps):

$$-\frac{1}{6} \operatorname{ArcTanh} [\operatorname{Cosh} [x]] - \frac{1}{6} \operatorname{ArcTanh} [2 \operatorname{Cosh} [x]] + \frac{\operatorname{ArcTanh} \left[ \frac{2 \operatorname{Cosh} [x]}{\sqrt{3}} \right]}{2\sqrt{3}}$$

Result (type 3, 91 leaves):

$$\frac{1}{12} \left( 2\sqrt{3} \operatorname{ArcTanh} \left[ \frac{2 - i \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] + 2\sqrt{3} \operatorname{ArcTanh} \left[ \frac{2 + i \operatorname{Tanh} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] - 2 \operatorname{Log} \left[ \operatorname{Cosh} \left[ \frac{x}{2} \right] \right] + \operatorname{Log} [1 - 2 \operatorname{Cosh} [x]] - \operatorname{Log} [1 + 2 \operatorname{Cosh} [x]] + 2 \operatorname{Log} \left[ \operatorname{Sinh} \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 254:** Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cosh} [a + bx] \operatorname{Sinh} [a + bx] dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\operatorname{Sinh} [a + bx]^2}{2b}$$

Result (type 3, 37 leaves):

$$\frac{1}{2} \left( \frac{\operatorname{Cosh} [2a] \operatorname{Cosh} [2bx]}{2b} + \frac{\operatorname{Sinh} [2a] \operatorname{Sinh} [2bx]}{2b} \right)$$

**Problem 337:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x \operatorname{Tanh} [a + bx] dx$$

Optimal (type 4, 45 leaves, 4 steps):

$$-\frac{x^2}{2} + \frac{x \operatorname{Log}[1 + e^{2(a+bx)}]}{b} + \frac{\operatorname{PolyLog}[2, -e^{2(a+bx)}]}{2b^2}$$

Result (type 4, 197 leaves):

$$\begin{aligned} & - \left( \left( \operatorname{Csch}[a] \left( -b^2 e^{-\operatorname{ArcTanh}[\operatorname{Coth}[a]]} x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth}[a]^2}} \right. \right. \right. \\ & \quad \left. \left. \left. \operatorname{Coth}[a] (-bx (-\pi + 2i \operatorname{ArcTanh}[\operatorname{Coth}[a])) - \pi \operatorname{Log}[1 + e^{2bx}] - 2(i bx + i \operatorname{ArcTanh}[\operatorname{Coth}[a]]) \operatorname{Log}[1 - e^{2i(i bx + i \operatorname{ArcTanh}[\operatorname{Coth}[a])}]] + \right. \right. \right. \\ & \quad \left. \left. \left. \pi \operatorname{Log}[\operatorname{Cosh}[bx]] + 2i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \operatorname{Log}[i \operatorname{Sinh}[bx + \operatorname{ArcTanh}[\operatorname{Coth}[a]]]] + i \operatorname{PolyLog}[2, e^{2i(i bx + i \operatorname{ArcTanh}[\operatorname{Coth}[a])}]] \right) \right) \right) \\ & \left. \operatorname{Sech}[a] \right) / \left( 2b^2 \sqrt{\operatorname{Csch}[a]^2 (-\operatorname{Cosh}[a]^2 + \operatorname{Sinh}[a]^2)} \right) + \frac{1}{2} x^2 \operatorname{Tanh}[a] \end{aligned}$$

**Problem 349: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int x^3 \operatorname{Sech}[a + bx]^2 \operatorname{Tanh}[a + bx] dx$$

Optimal (type 4, 83 leaves, 6 steps):

$$\frac{3x^2}{2b^2} - \frac{3x \operatorname{Log}[1 + e^{2(a+bx)}]}{b^3} - \frac{3 \operatorname{PolyLog}[2, -e^{2(a+bx)}]}{2b^4} - \frac{x^3 \operatorname{Sech}[a + bx]^2}{2b} + \frac{3x^2 \operatorname{Tanh}[a + bx]}{2b^2}$$

Result (type 4, 228 leaves):

$$\begin{aligned} & - \frac{x^3 \operatorname{Sech}[a + bx]^2}{2b} + \left( 3 \operatorname{Csch}[a] \left( -b^2 e^{-\operatorname{ArcTanh}[\operatorname{Coth}[a]]} x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth}[a]^2}} \right. \right. \\ & \quad \left. \left. \left. \operatorname{Coth}[a] (-bx (-\pi + 2i \operatorname{ArcTanh}[\operatorname{Coth}[a])) - \pi \operatorname{Log}[1 + e^{2bx}] - 2(i bx + i \operatorname{ArcTanh}[\operatorname{Coth}[a]]) \operatorname{Log}[1 - e^{2i(i bx + i \operatorname{ArcTanh}[\operatorname{Coth}[a])}]] + \right. \right. \right. \\ & \quad \left. \left. \left. \pi \operatorname{Log}[\operatorname{Cosh}[bx]] + 2i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \operatorname{Log}[i \operatorname{Sinh}[bx + \operatorname{ArcTanh}[\operatorname{Coth}[a]]]] + i \operatorname{PolyLog}[2, e^{2i(i bx + i \operatorname{ArcTanh}[\operatorname{Coth}[a])}]] \right) \right) \right) \operatorname{Sech}[a] \Big/ \\ & \left( 2b^4 \sqrt{\operatorname{Csch}[a]^2 (-\operatorname{Cosh}[a]^2 + \operatorname{Sinh}[a]^2)} \right) + \frac{3x^2 \operatorname{Sech}[a] \operatorname{Sech}[a + bx] \operatorname{Sinh}[bx]}{2b^2} \end{aligned}$$

**Problem 358: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Sinh}[a + bx] \operatorname{Tanh}[a + bx] dx$$

Optimal (type 4, 77 leaves, 8 steps):

$$-\frac{2 \times \text{ArcTan}\left[e^{a+bx}\right]}{b} - \frac{\text{Cosh}[a+bx]}{b^2} + \frac{i \text{PolyLog}\left[2, -i e^{a+bx}\right]}{b^2} - \frac{i \text{PolyLog}\left[2, i e^{a+bx}\right]}{b^2} + \frac{x \text{Sinh}[a+bx]}{b}$$

Result (type 4, 212 leaves):

$$\frac{1}{b^2} \left( \left( -i a + \frac{\pi}{2} - i b x \right) \left( \text{Log}\left[1 - e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] - \text{Log}\left[1 + e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] \right) - \left( -i a + \frac{\pi}{2} \right) \text{Log}\left[\text{Tan}\left[\frac{1}{2} \left( -i a + \frac{\pi}{2} - i b x \right)\right]\right] \right) + i \left( \text{PolyLog}\left[2, -e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] - \text{PolyLog}\left[2, e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] \right) + \frac{\text{Cosh}[bx] \left( -\text{Cosh}[a] + b x \text{Sinh}[a] \right)}{b^2} + \frac{\left( b x \text{Cosh}[a] - \text{Sinh}[a] \right) \text{Sinh}[bx]}{b^2}$$

**Problem 364:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x^2 \text{Tanh}[a+bx]^2 dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$-\frac{x^2}{b} + \frac{x^3}{3} + \frac{2 x \text{Log}\left[1 + e^{2(a+bx)}\right]}{b^2} + \frac{\text{PolyLog}\left[2, -e^{2(a+bx)}\right]}{b^3} - \frac{x^2 \text{Tanh}[a+bx]}{b}$$

Result (type 4, 213 leaves):

$$\frac{x^3}{3} - \left( \text{Csch}[a] \left( -b^2 e^{-\text{ArcTanh}[\text{Coth}[a]]} x^2 + \frac{1}{\sqrt{1 - \text{Coth}[a]^2}} \right) \right. \\ \left. i \text{Coth}[a] \left( -b x \left( -\pi + 2 i \text{ArcTanh}[\text{Coth}[a]] \right) - \pi \text{Log}\left[1 + e^{2bx}\right] - 2 \left( i b x + i \text{ArcTanh}[\text{Coth}[a]] \right) \text{Log}\left[1 - e^{2i \left( i b x + i \text{ArcTanh}[\text{Coth}[a]] \right)}\right] \right) + \right. \\ \left. \pi \text{Log}[\text{Cosh}[bx]] + 2 i \text{ArcTanh}[\text{Coth}[a]] \text{Log}\left[ i \text{Sinh}[bx + \text{ArcTanh}[\text{Coth}[a]]] \right] + i \text{PolyLog}\left[2, e^{2i \left( i b x + i \text{ArcTanh}[\text{Coth}[a]] \right)}\right] \right) \text{Sech}[a] \Bigg/ \\ \left( b^3 \sqrt{\text{Csch}[a]^2 \left( -\text{Cosh}[a]^2 + \text{Sinh}[a]^2 \right)} \right) - \frac{x^2 \text{Sech}[a] \text{Sech}[a+bx] \text{Sinh}[bx]}{b}$$

**Problem 393:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x \text{Tanh}[a+bx]^3 dx$$

Optimal (type 4, 82 leaves, 7 steps):

$$\frac{x}{2b} - \frac{x^2}{2} + \frac{x \text{Log}\left[1 + e^{2(a+bx)}\right]}{b} + \frac{\text{PolyLog}\left[2, -e^{2(a+bx)}\right]}{2b^2} - \frac{\text{Tanh}[a+bx]}{2b^2} - \frac{x \text{Tanh}[a+bx]^2}{2b}$$



Result (type 4, 232 leaves):

$$\frac{x \operatorname{Sech}[a + b x]^2}{2 b} - \left( \operatorname{Csch}[a] \left( -b^2 e^{-\operatorname{ArcTanh}[\operatorname{Coth}[a]]} x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth}[a]^2}} \right. \right. \\ \left. \left. + i \operatorname{Coth}[a] (-b x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[a]]) - \pi \operatorname{Log}[1 + e^{2 b x}] - 2 (i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]]) \operatorname{Log}[1 - e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]])}]) \right. \right. \\ \left. \left. + \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \operatorname{Log}[i \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Coth}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]])}] \right) \operatorname{Sech}[a] \right) / \\ \left( 2 b^2 \sqrt{\operatorname{Csch}[a]^2 (-\operatorname{Cosh}[a]^2 + \operatorname{Sinh}[a]^2)} \right) - \frac{\operatorname{Sech}[a] \operatorname{Sech}[a + b x] \operatorname{Sinh}[b x]}{2 b^2} + \frac{1}{2} \\ x^2 \\ \operatorname{Tanh}[a]$$

**Problem 400:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x \operatorname{Coth}[a + b x] dx$$

Optimal (type 4, 45 leaves, 4 steps):

$$-\frac{x^2}{2} + \frac{x \operatorname{Log}[1 - e^{2(a+bx)}]}{b} + \frac{\operatorname{PolyLog}[2, e^{2(a+bx)}]}{2 b^2}$$

Result (type 4, 148 leaves):

$$\frac{1}{2 b^2} \left( i b \pi x + b^2 x^2 \operatorname{Coth}[a] - i \pi \operatorname{Log}[1 + e^{2 b x}] + 2 b x \operatorname{Log}[1 - e^{-2(b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]])}] \right) + \\ i \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 \operatorname{ArcTanh}[\operatorname{Tanh}[a]] (b x + \operatorname{Log}[1 - e^{-2(b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]])}]) - \operatorname{Log}[i \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]]]] - \\ \operatorname{PolyLog}[2, e^{-2(b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]])}] - b^2 e^{-\operatorname{ArcTanh}[\operatorname{Tanh}[a]]} x^2 \operatorname{Coth}[a] \sqrt{\operatorname{Sech}[a]^2}$$

**Problem 420:** Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \operatorname{Cosh}[x]^2 \operatorname{Coth}[x]^2 dx$$

Optimal (type 4, 102 leaves, 12 steps):

$$\frac{3 x^2}{8} - x^3 + \frac{3 x^4}{8} - \frac{3 \operatorname{Cosh}[x]^2}{8} - \frac{3}{4} x^2 \operatorname{Cosh}[x]^2 - x^3 \operatorname{Coth}[x] + 3 x^2 \operatorname{Log}[1 - e^{2 x}] + \\ 3 x \operatorname{PolyLog}[2, e^{2 x}] - \frac{3}{2} \operatorname{PolyLog}[3, e^{2 x}] + \frac{3}{4} x \operatorname{Cosh}[x] \operatorname{Sinh}[x] + \frac{1}{2} x^3 \operatorname{Cosh}[x] \operatorname{Sinh}[x]$$

Result (type 4, 94 leaves):

$$\frac{1}{16} \left( 2 i \pi^3 - 16 x^3 + 6 x^4 - 3 \operatorname{Cosh}[2 x] - 6 x^2 \operatorname{Cosh}[2 x] - 16 x^3 \operatorname{Coth}[x] + \right. \\ \left. 48 x^2 \operatorname{Log}[1 - e^{2x}] + 48 x \operatorname{PolyLog}[2, e^{2x}] - 24 \operatorname{PolyLog}[3, e^{2x}] + 6 x \operatorname{Sinh}[2 x] + 4 x^3 \operatorname{Sinh}[2 x] \right)$$

**Problem 422: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 \operatorname{Cosh}[x]^2 \operatorname{Coth}[x]^3 dx$$

Optimal (type 4, 96 leaves, 19 steps):

$$\frac{3 x^2}{4} - \frac{2 x^3}{3} - x \operatorname{Coth}[x] - \frac{1}{2} x^2 \operatorname{Coth}[x]^2 + 2 x^2 \operatorname{Log}[1 - e^{2x}] + \operatorname{Log}[\operatorname{Sinh}[x]] + \\ 2 x \operatorname{PolyLog}[2, e^{2x}] - \operatorname{PolyLog}[3, e^{2x}] - \frac{1}{2} x \operatorname{Cosh}[x] \operatorname{Sinh}[x] + \frac{\operatorname{Sinh}[x]^2}{4} + \frac{1}{2} x^2 \operatorname{Sinh}[x]^2$$

Result (type 4, 98 leaves):

$$\frac{i \pi^3}{12} - \frac{2 x^3}{3} + \frac{1}{8} \operatorname{Cosh}[2 x] + \frac{1}{4} x^2 \operatorname{Cosh}[2 x] - x \operatorname{Coth}[x] - \frac{1}{2} x^2 \operatorname{CsCh}[x]^2 + \\ 2 x^2 \operatorname{Log}[1 - e^{2x}] + \operatorname{Log}[\operatorname{Sinh}[x]] + 2 x \operatorname{PolyLog}[2, e^{2x}] - \operatorname{PolyLog}[3, e^{2x}] - \frac{1}{4} x \operatorname{Sinh}[2 x]$$

**Problem 426: Result more than twice size of optimal antiderivative.**

$$\int x^2 \operatorname{Coth}[a + b x] \operatorname{CsCh}[a + b x] dx$$

Optimal (type 4, 59 leaves, 6 steps):

$$-\frac{4 x \operatorname{ArcTanh}[e^{a+bx}]}{b^2} - \frac{x^2 \operatorname{CsCh}[a + b x]}{b} - \frac{2 \operatorname{PolyLog}[2, -e^{a+bx}]}{b^3} + \frac{2 \operatorname{PolyLog}[2, e^{a+bx}]}{b^3}$$

Result (type 4, 133 leaves):

$$-\frac{1}{b^3} \left( b^2 x^2 \operatorname{CsCh}[a + b x] - 2 a \operatorname{Log}[1 - e^{-a-bx}] - 2 b x \operatorname{Log}[1 - e^{-a-bx}] + 2 a \operatorname{Log}[1 + e^{-a-bx}] + \right. \\ \left. 2 b x \operatorname{Log}[1 + e^{-a-bx}] + 2 a \operatorname{Log}\left[\operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]\right] - 2 \operatorname{PolyLog}[2, -e^{-a-bx}] + 2 \operatorname{PolyLog}[2, e^{-a-bx}] \right)$$

### Problem 427: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Coth}[a + b x] \operatorname{Csch}[a + b x] dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[a + b x]]}{b^2} - \frac{x \operatorname{Csch}[a + b x]}{b}$$

Result (type 3, 114 leaves):

$$-\frac{x \operatorname{Csch}[a]}{b} - \frac{\operatorname{Log}[\operatorname{Cosh}[\frac{a}{2} + \frac{bx}{2}]]}{b^2} + \frac{\operatorname{Log}[\operatorname{Sinh}[\frac{a}{2} + \frac{bx}{2}]]}{b^2} + \frac{x \operatorname{Csch}[\frac{a}{2}] \operatorname{Csch}[\frac{a}{2} + \frac{bx}{2}] \operatorname{Sinh}[\frac{bx}{2}]}{2b} + \frac{x \operatorname{Sech}[\frac{a}{2}] \operatorname{Sech}[\frac{a}{2} + \frac{bx}{2}] \operatorname{Sinh}[\frac{bx}{2}]}{2b}$$

### Problem 433: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{Coth}[a + b x]^2 dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$-\frac{x^2}{b} + \frac{x^3}{3} - \frac{x^2 \operatorname{Coth}[a + b x]}{b} + \frac{2x \operatorname{Log}[1 - e^{2(a+bx)}]}{b^2} + \frac{\operatorname{PolyLog}[2, e^{2(a+bx)}]}{b^3}$$

Result (type 4, 211 leaves):

$$\frac{x^3}{3} + \frac{x^2 \operatorname{Csch}[a] \operatorname{Csch}[a + b x] \operatorname{Sinh}[b x]}{b} + \left( \operatorname{Csch}[a] \operatorname{Sech}[a] \left( -b^2 e^{-\operatorname{ArcTanh}[\operatorname{Tanh}[a]]} x^2 + \frac{1}{\sqrt{1 - \operatorname{Tanh}[a]^2}} i (-b x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]]) - \pi \operatorname{Log}[1 + e^{2bx}]) - \right. \right. \\ \left. \left. 2 (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a]]) \operatorname{Log}[1 - e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a])}] + \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \right. \right. \\ \left. \left. \operatorname{Log}[i \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]]] + i \operatorname{PolyLog}[2, e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a])}]] \operatorname{Tanh}[a] \right) \right) / \left( b^3 \sqrt{\operatorname{Sech}[a]^2 (\operatorname{Cosh}[a]^2 - \operatorname{Sinh}[a]^2)} \right)$$

### Problem 440: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{Cosh}[a + b x] \operatorname{Coth}[a + b x]^2 dx$$

Optimal (type 4, 95 leaves, 10 steps):

$$-\frac{4 x \operatorname{ArcTanh}\left[e^{a+b x}\right]}{b^2}-\frac{2 x \operatorname{Cosh}[a+b x]}{b^2}-\frac{x^2 \operatorname{Csch}[a+b x]}{b}-\frac{2 \operatorname{PolyLog}\left[2,-e^{a+b x}\right]}{b^3}+\frac{2 \operatorname{PolyLog}\left[2, e^{a+b x}\right]}{b^3}+\frac{2 \operatorname{Sinh}[a+b x]}{b^3}+\frac{x^2 \operatorname{Sinh}[a+b x]}{b}$$

Result (type 4, 230 leaves):

$$\frac{1}{4 b^3} \operatorname{Csch}\left[\frac{1}{2}(a+b x)\right] \operatorname{Sech}\left[\frac{1}{2}(a+b x)\right] \\ \left(-2-3 b^2 x^2+2 \operatorname{Cosh}\left[2(a+b x)\right]+b^2 x^2 \operatorname{Cosh}\left[2(a+b x)\right]+4 a \operatorname{Log}\left[1-e^{-a-b x}\right] \operatorname{Sinh}[a+b x]+4 b x \operatorname{Log}\left[1-e^{-a-b x}\right] \operatorname{Sinh}[a+b x]-\right. \\ \left.4 a \operatorname{Log}\left[1+e^{-a-b x}\right] \operatorname{Sinh}[a+b x]-4 b x \operatorname{Log}\left[1+e^{-a-b x}\right] \operatorname{Sinh}[a+b x]-4 a \operatorname{Log}\left[\operatorname{Tanh}\left[\frac{1}{2}(a+b x)\right]\right] \operatorname{Sinh}[a+b x]+4 \operatorname{PolyLog}\left[2,-e^{-a-b x}\right] \operatorname{Sinh}[a+b x]-4 \operatorname{PolyLog}\left[2, e^{-a-b x}\right] \operatorname{Sinh}[a+b x]-2 b x \operatorname{Sinh}\left[2(a+b x)\right]\right)$$

**Problem 442: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cosh}[a+b x] \operatorname{Coth}[a+b x]^2 dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$-\frac{\operatorname{Csch}[a+b x]}{b}+\frac{\operatorname{Sinh}[a+b x]}{b}$$

Result (type 3, 45 leaves):

$$-\frac{\operatorname{Coth}\left[\frac{1}{2}(a+b x)\right]}{2 b}+\frac{\operatorname{Sinh}[a+b x]}{b}+\frac{\operatorname{Tanh}\left[\frac{1}{2}(a+b x)\right]}{2 b}$$

**Problem 446: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int x^3 \operatorname{Coth}[a+b x] \operatorname{Csch}[a+b x]^2 dx$$

Optimal (type 4, 83 leaves, 6 steps):

$$-\frac{3 x^2}{2 b^2}-\frac{3 x^2 \operatorname{Coth}[a+b x]}{2 b^2}-\frac{x^3 \operatorname{Csch}[a+b x]^2}{2 b}+\frac{3 x \operatorname{Log}\left[1-e^{2(a+b x)}\right]}{b^3}+\frac{3 \operatorname{PolyLog}\left[2, e^{2(a+b x)}\right]}{2 b^4}$$

Result (type 4, 228 leaves):

$$\begin{aligned}
& -\frac{x^3 \operatorname{Csch}[a + b x]^2}{2 b} + \frac{3 x^2 \operatorname{Csch}[a] \operatorname{Csch}[a + b x] \operatorname{Sinh}[b x]}{2 b^2} + \\
& \left( 3 \operatorname{Csch}[a] \operatorname{Sech}[a] \left( -b^2 e^{-\operatorname{ArcTanh}[\operatorname{Tanh}[a]]} x^2 + \frac{1}{\sqrt{1 - \operatorname{Tanh}[a]^2}} i (-b x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]]) - \pi \operatorname{Log}[1 + e^{2 b x}] - \right. \right. \\
& \quad \left. \left. 2 (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a]]) \operatorname{Log}[1 - e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a])}] \right] + \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \right. \\
& \quad \left. \left. \operatorname{Log}[i \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]]] + i \operatorname{PolyLog}[2, e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a])}]] \operatorname{Tanh}[a] \right) \right) \Bigg/ \left( 2 b^4 \sqrt{\operatorname{Sech}[a]^2 (\operatorname{Cosh}[a]^2 - \operatorname{Sinh}[a]^2)} \right)
\end{aligned}$$

**Problem 456: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Coth}[a + b x]^2 \operatorname{Csch}[a + b x] dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[a + b x]]}{2 b} - \frac{\operatorname{Coth}[a + b x] \operatorname{Csch}[a + b x]}{2 b}$$

Result (type 3, 75 leaves):

$$-\frac{\operatorname{Csch}\left[\frac{1}{2}(a + b x)\right]^2}{8 b} - \frac{\operatorname{Log}[\operatorname{Cosh}\left[\frac{1}{2}(a + b x)\right]]}{2 b} + \frac{\operatorname{Log}[\operatorname{Sinh}\left[\frac{1}{2}(a + b x)\right]]}{2 b} - \frac{\operatorname{Sech}\left[\frac{1}{2}(a + b x)\right]^2}{8 b}$$

**Problem 462: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int x \operatorname{Coth}[a + b x]^3 dx$$

Optimal (type 4, 82 leaves, 7 steps):

$$\frac{x}{2 b} - \frac{x^2}{2} - \frac{\operatorname{Coth}[a + b x]}{2 b^2} - \frac{x \operatorname{Coth}[a + b x]^2}{2 b} + \frac{x \operatorname{Log}[1 - e^{2(a + b x)}]}{b} + \frac{\operatorname{PolyLog}[2, e^{2(a + b x)}]}{2 b^2}$$

Result (type 4, 232 leaves):

$$\frac{1}{2} x^2 \operatorname{Coth}[a] - \frac{x \operatorname{Csch}[a + b x]^2}{2 b} + \frac{\operatorname{Csch}[a] \operatorname{Csch}[a + b x] \operatorname{Sinh}[b x]}{2 b^2} +$$

$$\left( \operatorname{Csch}[a] \operatorname{Sech}[a] \left( -b^2 e^{-\operatorname{ArcTanh}[\operatorname{Tanh}[a]]} x^2 + \frac{1}{\sqrt{1 - \operatorname{Tanh}[a]^2}} i (-b x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]]) - \pi \operatorname{Log}[1 + e^{2 b x}] - \right. \right.$$

$$2 (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a]]) \operatorname{Log}[1 - e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a])}] + \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]]$$

$$\left. \left. \operatorname{Log}[i \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]]] + i \operatorname{PolyLog}[2, e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a])}]\right) \operatorname{Tanh}[a] \right) \right) / \left( 2 b^2 \sqrt{\operatorname{Sech}[a]^2 (\operatorname{Cosh}[a]^2 - \operatorname{Sinh}[a]^2)} \right)$$

**Problem 470: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csch}[a + b x] \operatorname{Sech}[a + b x] dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{\operatorname{Log}[\operatorname{Tanh}[a + b x]]}{b}$$

Result (type 3, 31 leaves):

$$2 \left( -\frac{\operatorname{Log}[\operatorname{Cosh}[a + b x]]}{2 b} + \frac{\operatorname{Log}[\operatorname{Sinh}[a + b x]]}{2 b} \right)$$

**Problem 485: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Csch}[a + b x] \operatorname{Sech}[a + b x]^3}{x} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{\operatorname{Csch}[a + b x] \operatorname{Sech}[a + b x]^3}{x}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 491: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csch}[a + b x]^2 \operatorname{Sech}[a + b x] dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{\text{ArcTan}[\text{Sinh}[a + b x]]}{b} - \frac{\text{Csch}[a + b x]}{b}$$

Result (type 3, 51 leaves):

$$-\frac{2 \text{ArcTan}\left[\text{Tanh}\left[\frac{1}{2}(a + b x)\right]\right]}{b} - \frac{\text{Coth}\left[\frac{1}{2}(a + b x)\right]}{2 b} + \frac{\text{Tanh}\left[\frac{1}{2}(a + b x)\right]}{2 b}$$

**Problem 505: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Csch}[a + b x]^2 \text{Sech}[a + b x]^3}{x} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Csch}[a + b x]^2 \text{Sech}[a + b x]^3}{x}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 512: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Csch}[a + b x]^3 \text{Sech}[a + b x]}{x} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Csch}[a + b x]^3 \text{Sech}[a + b x]}{x}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 516: Result more than twice size of optimal antiderivative.**

$$\int x^2 \text{Csch}[a + b x]^3 \text{Sech}[a + b x]^2 dx$$

Optimal (type 4, 197 leaves, 29 steps):

$$\frac{4 x \operatorname{ArcTan}\left[e^{a+b x}\right]}{b^2} + \frac{3 x^2 \operatorname{ArcTanh}\left[e^{a+b x}\right]}{b} - \frac{\operatorname{ArcTanh}\left[\operatorname{Cosh}\left[a+b x\right]\right]}{b^3} - \frac{x \operatorname{Csch}\left[a+b x\right]}{b^2} + \frac{3 x \operatorname{PolyLog}\left[2,-e^{a+b x}\right]}{b^2} - \frac{2 i \operatorname{PolyLog}\left[2,-i e^{a+b x}\right]}{b^3} +$$

$$\frac{2 i \operatorname{PolyLog}\left[2,i e^{a+b x}\right]}{b^3} - \frac{3 x \operatorname{PolyLog}\left[2,e^{a+b x}\right]}{b^2} - \frac{3 \operatorname{PolyLog}\left[3,-e^{a+b x}\right]}{b^3} + \frac{3 \operatorname{PolyLog}\left[3,e^{a+b x}\right]}{b^3} - \frac{3 x^2 \operatorname{Sech}\left[a+b x\right]}{2 b} - \frac{x^2 \operatorname{Csch}\left[a+b x\right]^2 \operatorname{Sech}\left[a+b x\right]}{2 b}$$

Result (type 4, 425 leaves):

$$-\frac{x \operatorname{Csch}\left[a\right]}{b^2} - \frac{x^2 \operatorname{Csch}\left[\frac{a}{2} + \frac{b x}{2}\right]^2}{8 b} - \frac{1}{b^3}$$

$$2 \left( \left( -i a + \frac{\pi}{2} - i b x \right) \left( \operatorname{Log}\left[1 - e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] \right) - \operatorname{Log}\left[1 + e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] \right) - \left( -i a + \frac{\pi}{2} \right) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} \left( -i a + \frac{\pi}{2} - i b x \right)\right]\right] +$$

$$i \left( \operatorname{PolyLog}\left[2,-e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] - \operatorname{PolyLog}\left[2,e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] \right) - \frac{1}{2 b^3} \left( 4 \operatorname{ArcTanh}\left[e^{a+b x}\right] + 3 b^2 x^2 \operatorname{Log}\left[1 - e^{a+b x}\right] - \right.$$

$$\left. 3 b^2 x^2 \operatorname{Log}\left[1 + e^{a+b x}\right] - 6 b x \operatorname{PolyLog}\left[2,-e^{a+b x}\right] + 6 b x \operatorname{PolyLog}\left[2,e^{a+b x}\right] + 6 \operatorname{PolyLog}\left[3,-e^{a+b x}\right] - 6 \operatorname{PolyLog}\left[3,e^{a+b x}\right] \right) -$$

$$\frac{x^2 \operatorname{Sech}\left[\frac{a}{2} + \frac{b x}{2}\right]^2}{8 b} - \frac{x^2 \operatorname{Sech}\left[a+b x\right]}{b} + \frac{x \operatorname{Csch}\left[\frac{a}{2}\right] \operatorname{Csch}\left[\frac{a}{2} + \frac{b x}{2}\right] \operatorname{Sinh}\left[\frac{b x}{2}\right]}{2 b^2} + \frac{x \operatorname{Sech}\left[\frac{a}{2}\right] \operatorname{Sech}\left[\frac{a}{2} + \frac{b x}{2}\right] \operatorname{Sinh}\left[\frac{b x}{2}\right]}{2 b^2}$$

Problem 519: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Csch}\left[a+b x\right]^3 \operatorname{Sech}\left[a+b x\right]^2}{x} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\operatorname{Csch}\left[a+b x\right]^3 \operatorname{Sech}\left[a+b x\right]^2}{x}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 529: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x \operatorname{Cosh}\left[a+b x\right]^{3/2} \operatorname{Sinh}\left[a+b x\right] dx$$

Optimal (type 4, 64 leaves, 3 steps):

$$\frac{2 x \operatorname{Cosh}\left[a+b x\right]^{5/2}}{5 b} + \frac{12 i \operatorname{EllipticE}\left[\frac{1}{2} i \left(a+b x\right), 2\right]}{25 b^2} - \frac{4 \operatorname{Cosh}\left[a+b x\right]^{3/2} \operatorname{Sinh}\left[a+b x\right]}{25 b^2}$$



Result (type 5, 142 leaves):

$$\frac{1}{50 \sqrt{2} b^2 \sqrt{e^{-a-bx} + e^{a+bx}}} e^{-3(a+bx)} \left( (1 + e^{2(a+bx)}) (2 + 5bx + 2e^{2(a+bx)}(-12 + 5bx)) + e^{4(a+bx)}(-2 + 5bx) \right) + 48 e^{2(a+bx)} \sqrt{1 + e^{2(a+bx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2(a+bx)}\right]$$

**Problem 531:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x \operatorname{Sinh}[a+bx]}{\sqrt{\operatorname{Cosh}[a+bx]}} dx$$

Optimal (type 4, 37 leaves, 2 steps):

$$\frac{2x \sqrt{\operatorname{Cosh}[a+bx]}}{b} + \frac{4i \operatorname{EllipticE}\left[\frac{1}{2}i(a+bx), 2\right]}{b^2}$$

Result (type 5, 109 leaves):

$$\frac{1}{b^2 \sqrt{\operatorname{Cosh}[a+bx]}} (\operatorname{Cosh}[a+bx] - \operatorname{Sinh}[a+bx]) \left( 4 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\operatorname{Cosh}[2(a+bx)] - \operatorname{Sinh}[2(a+bx)]\right] \sqrt{1 + \operatorname{Cosh}[2(a+bx)] + \operatorname{Sinh}[2(a+bx)]} + (-2+bx) (1 + \operatorname{Cosh}[2(a+bx)] + \operatorname{Sinh}[2(a+bx)]) \right)$$

**Problem 540:** Result unnecessarily involves higher level functions.

$$\int x \sqrt{\operatorname{Sech}[a+bx]} \operatorname{Sinh}[a+bx] dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$\frac{2x}{b \sqrt{\operatorname{Sech}[a+bx]}} + \frac{4i \sqrt{\operatorname{Cosh}[a+bx]} \operatorname{EllipticE}\left[\frac{1}{2}i(a+bx), 2\right] \sqrt{\operatorname{Sech}[a+bx]}}{b^2}$$

Result (type 5, 100 leaves):

$$\frac{1}{b^2} \sqrt{2} e^{-a-bx} \sqrt{\frac{e^{a+bx}}{1 + e^{2(a+bx)}}} \left( (1 + e^{2(a+bx)}) (-2 + bx) + 4 \sqrt{1 + e^{2(a+bx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2(a+bx)}\right] \right)$$

**Problem 542: Result unnecessarily involves higher level functions.**

$$\int \frac{x \operatorname{Sinh}[a + b x]}{\operatorname{Sech}[a + b x]^{3/2}} dx$$

Optimal (type 4, 84 leaves, 4 steps):

$$\frac{2 x}{5 b \operatorname{Sech}[a + b x]^{5/2}} + \frac{12 i \sqrt{\operatorname{Cosh}[a + b x]} \operatorname{EllipticE}\left[\frac{1}{2} i (a + b x), 2\right] \sqrt{\operatorname{Sech}[a + b x]}}{25 b^2} - \frac{4 \operatorname{Sinh}[a + b x]}{25 b^2 \operatorname{Sech}[a + b x]^{3/2}}$$

Result (type 5, 125 leaves):

$$\frac{1}{100 b^2} e^{-3(a+bx)} \left( (1 + e^{2(a+bx)}) (2 + 5bx + 2e^{2(a+bx)}(-12 + 5bx)) + e^{4(a+bx)}(-2 + 5bx) \right) + 48 e^{2(a+bx)} \sqrt{1 + e^{2(a+bx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2(a+bx)}\right] \Bigg/ \sqrt{\operatorname{Sech}[a + b x]}$$

**Problem 545: Result unnecessarily involves higher level functions.**

$$\int x \operatorname{Cosh}[a + b x] \operatorname{Sinh}[a + b x]^{3/2} dx$$

Optimal (type 4, 98 leaves, 4 steps):

$$-\frac{12 i \operatorname{EllipticE}\left[\frac{1}{2} \left(i a - \frac{\pi}{2} + i b x\right), 2\right] \sqrt{\operatorname{Sinh}[a + b x]}}{25 b^2 \sqrt{i \operatorname{Sinh}[a + b x]}} - \frac{4 \operatorname{Cosh}[a + b x] \operatorname{Sinh}[a + b x]^{3/2}}{25 b^2} + \frac{2 x \operatorname{Sinh}[a + b x]^{5/2}}{5 b}$$

Result (type 5, 143 leaves):

$$\left( e^{-3(a+bx)} \left( (-1 + e^{2(a+bx)}) (2 + 5bx + e^{2(a+bx)}(24 - 10bx)) + e^{4(a+bx)}(-2 + 5bx) \right) + 48 e^{2(a+bx)} \sqrt{1 - e^{2(a+bx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2(a+bx)}\right] \right) \Bigg/ \left( 50 \sqrt{2} b^2 \sqrt{-e^{-a-bx} + e^{a+bx}} \right)$$

**Problem 547: Result unnecessarily involves higher level functions.**

$$\int \frac{x \operatorname{Cosh}[a + b x]}{\sqrt{\operatorname{Sinh}[a + b x]}} dx$$

Optimal (type 4, 71 leaves, 3 steps):

$$\frac{2x \sqrt{\text{Sinh}[a+bx]}}{b} + \frac{4i \text{EllipticE}\left[\frac{1}{2}\left(ia - \frac{\pi}{2} + ibx\right), 2\right] \sqrt{\text{Sinh}[a+bx]}}{b^2 \sqrt{i \text{Sinh}[a+bx]}}$$

Result (type 5, 115 leaves):

$$\frac{1}{b^2 \sqrt{\text{Sinh}[a+bx]}} \left( -\text{Cosh}[a+bx] + \text{Sinh}[a+bx] \right) \left( -2(-2+bx) \text{Sinh}[a+bx] (\text{Cosh}[a+bx] + \text{Sinh}[a+bx]) + 4\sqrt{2} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \text{Cosh}[2(a+bx)] + \text{Sinh}[2(a+bx)]\right] \sqrt{-\text{Sinh}[a+bx] (\text{Cosh}[a+bx] + \text{Sinh}[a+bx])} \right)$$

**Problem 556: Result unnecessarily involves higher level functions.**

$$\int x \text{Cosh}[a+bx] \sqrt{\text{Csch}[a+bx]} dx$$

Optimal (type 4, 71 leaves, 3 steps):

$$\frac{2x}{b \sqrt{\text{Csch}[a+bx]}} + \frac{4i \text{EllipticE}\left[\frac{1}{2}\left(ia - \frac{\pi}{2} + ibx\right), 2\right]}{b^2 \sqrt{\text{Csch}[a+bx]} \sqrt{i \text{Sinh}[a+bx]}}$$

Result (type 5, 100 leaves):

$$\frac{1}{b^2} \sqrt{2} e^{-a-bx} \sqrt{\frac{e^{a+bx}}{-1+e^{2(a+bx)}}} \left( (-1+e^{2(a+bx)}) (-2+bx) - 4\sqrt{1-e^{2(a+bx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2(a+bx)}\right] \right)$$

**Problem 558: Result unnecessarily involves higher level functions.**

$$\int \frac{x \text{Cosh}[a+bx]}{\text{Csch}[a+bx]^{3/2}} dx$$

Optimal (type 4, 98 leaves, 4 steps):

$$\frac{2x}{5b \text{Csch}[a+bx]^{5/2}} - \frac{4 \text{Cosh}[a+bx]}{25b^2 \text{Csch}[a+bx]^{3/2}} - \frac{12i \text{EllipticE}\left[\frac{1}{2}\left(ia - \frac{\pi}{2} + ibx\right), 2\right]}{25b^2 \sqrt{\text{Csch}[a+bx]} \sqrt{i \text{Sinh}[a+bx]}}$$

Result (type 5, 111 leaves):

$$\frac{1}{50b^2 \sqrt{\text{Csch}[a+bx]}} e^{-2(a+bx)} \left( 2+5bx + e^{2(a+bx)} (24-10bx) + e^{4(a+bx)} (-2+5bx) - \frac{48e^{2(a+bx)} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2(a+bx)}\right]}{\sqrt{1-e^{2(a+bx)}}} \right)$$

### Problem 563: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\text{Cosh}[x] \text{Coth}[x]} \, dx$$

Optimal (type 3, 13 leaves, 3 steps):

$$2 \sqrt{\text{Cosh}[x] \text{Coth}[x]} \text{Tanh}[x]$$

Result (type 3, 35 leaves):

$$\frac{2 \sqrt{\text{Cosh}[x] \text{Coth}[x]} \left( -1 + (-\text{Sinh}[x]^2)^{1/4} \right) \text{Tanh}[x]}{(-\text{Sinh}[x]^2)^{1/4}}$$

### Problem 584: Result more than twice size of optimal antiderivative.

$$\int (a \text{Cosh}[x] + b \text{Sinh}[x])^5 \, dx$$

Optimal (type 3, 61 leaves, 3 steps):

$$(a^2 - b^2)^2 (b \text{Cosh}[x] + a \text{Sinh}[x]) + \frac{2}{3} (a^2 - b^2) (b \text{Cosh}[x] + a \text{Sinh}[x])^3 + \frac{1}{5} (b \text{Cosh}[x] + a \text{Sinh}[x])^5$$

Result (type 3, 133 leaves):

$$\frac{1}{240} \left( 150 b (a^2 - b^2)^2 \text{Cosh}[x] - 25 b (-3 a^4 + 2 a^2 b^2 + b^4) \text{Cosh}[3 x] + 3 b (5 a^4 + 10 a^2 b^2 + b^4) \text{Cosh}[5 x] + 150 a (a^2 - b^2)^2 \text{Sinh}[x] + 25 a (a^4 + 2 a^2 b^2 - 3 b^4) \text{Sinh}[3 x] + 3 a (a^4 + 10 a^2 b^2 + 5 b^4) \text{Sinh}[5 x] \right)$$

### Problem 590: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a \text{Cosh}[x] + b \text{Sinh}[x]} \, dx$$

Optimal (type 4, 65 leaves, 2 steps):

$$\frac{2 i \text{EllipticE}\left[\frac{1}{2} (i x - \text{ArcTan}[a, -i b]), 2\right] \sqrt{a \text{Cosh}[x] + b \text{Sinh}[x]}}{\sqrt{\frac{a \text{Cosh}[x] + b \text{Sinh}[x]}{a^2 - b^2}}}$$

Result (type 5, 206 leaves):

$$\left( b (-a^2 + b^2) \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] + \right. \\ \left. \sqrt{-\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2} \left( 2a^3 \sqrt{1 - \frac{b^2}{a^2}} \operatorname{Cosh}[x] - 2a(a^2 - b^2) \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] + 2a^2b \sqrt{1 - \frac{b^2}{a^2}} \operatorname{Sinh}[x] + \right. \right. \\ \left. \left. a^2b \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] - b^3 \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] \right) \right) / \left( a b \sqrt{1 - \frac{b^2}{a^2}} \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]} \sqrt{-\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2} \right)$$

**Problem 591: Result unnecessarily involves higher level functions.**

$$\int (a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^{3/2} dx$$

Optimal (type 4, 103 leaves, 3 steps):

$$\frac{2}{3} (b \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]) \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]} - \frac{2i(a^2 - b^2) \operatorname{EllipticF}\left[\frac{1}{2}(ix - \operatorname{ArcTan}[a, -ib]), 2\right] \sqrt{\frac{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{\sqrt{a^2 - b^2}}}}{3 \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}}$$

Result (type 5, 92 leaves):

$$\frac{2}{3} \left( b \operatorname{Cosh}[x] - \sqrt{1 - \frac{a^2}{b^2}} b \sqrt{\operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, -\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right]^2\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right] + \right. \\ \left. a \operatorname{Sinh}[x] \right) \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}$$

**Problem 592: Result unnecessarily involves higher level functions.**

$$\int (a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^{5/2} dx$$

Optimal (type 4, 103 leaves, 3 steps):

$$\frac{2}{5} (b \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]) (a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^{3/2} - \frac{6i(a^2 - b^2) \operatorname{EllipticE}\left[\frac{1}{2}(ix - \operatorname{ArcTan}[a, -ib]), 2\right] \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}}{5 \sqrt{\frac{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{\sqrt{a^2 - b^2}}}}$$

Result (type 5, 193 leaves):

$$\left( (a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]) (6 a (a^2 - b^2) + 2 a b^2 \operatorname{Cosh}[2 x] + b (a^2 + b^2) \operatorname{Sinh}[2 x]) - \right. \\ \left. \left( 3 (a - b)^2 (a + b)^2 \left( b \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] + \sqrt{-\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2} \right. \right. \right. \\ \left. \left. \left( 2 a \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] - b \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] \right) \right) \right) / \left( a \sqrt{1 - \frac{b^2}{a^2}} \sqrt{-\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2} \right) / (5 b \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]})$$

**Problem 593: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}} dx$$

Optimal (type 4, 65 leaves, 2 steps):

$$\frac{2 i \operatorname{EllipticF}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[a, -i b]), 2\right] \sqrt{\frac{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{\sqrt{a^2 - b^2}}}}{\sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}}$$

Result (type 5, 81 leaves):

$$\frac{1}{\sqrt{1 - \frac{a^2}{b^2}} b} 2 \sqrt{\operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, -\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right]^2\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right] \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}$$

**Problem 594: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^{3/2}} dx$$

Optimal (type 4, 112 leaves, 3 steps):

$$\frac{2 (b \operatorname{Cosh}[x] + a \operatorname{Sinh}[x])}{(a^2 - b^2) \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}} + \frac{2 i \operatorname{EllipticE}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[a, -i b]), 2\right] \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}}{(a^2 - b^2) \sqrt{\frac{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{\sqrt{a^2 - b^2}}}}$$

Result (type 5, 148 leaves):

$$\left( b \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] - \sqrt{-\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2} \left( 2 a \sqrt{1 - \frac{b^2}{a^2}} \operatorname{Cosh}[x] - 2 a \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] + b \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right] \right) \right) / \left( a b \sqrt{1 - \frac{b^2}{a^2}} \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]} \sqrt{-\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{a}\right]\right]^2} \right)$$

**Problem 595: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^{5/2}} dx$$

Optimal (type 4, 116 leaves, 3 steps):

$$\frac{2 (b \operatorname{Cosh}[x] + a \operatorname{Sinh}[x])}{3 (a^2 - b^2) (a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^{3/2}} - \frac{2 i \operatorname{EllipticF}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[a, -i b]), 2\right] \sqrt{\frac{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{a^2 - b^2}}}{3 (a^2 - b^2) \sqrt{a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}}$$

Result (type 5, 133 leaves):

$$- \left( \left( 2 \sqrt{1 - \frac{a^2}{b^2}} b (b \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]) + \sqrt{\operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, -\operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right]^2\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right] (a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^2 \right) / \left( 3 \sqrt{1 - \frac{a^2}{b^2}} b (-a + b) (a + b) (a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^{3/2} \right) \right)$$

**Problem 648: Result more than twice size of optimal antiderivative.**

$$\int (a \operatorname{Coth}[x] + b \operatorname{Csch}[x]) dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-b \operatorname{ArcTanh}[\operatorname{Cosh}[x]] + a \operatorname{Log}[\operatorname{Sinh}[x]]$$

Result (type 3, 25 leaves):

$$-b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] + a \operatorname{Log}[\operatorname{Sinh}[x]]$$

**Problem 658: Result more than twice size of optimal antiderivative.**

$$\int (\operatorname{Coth}[x] + \operatorname{Csch}[x]) \, dx$$

Optimal (type 3, 9 leaves, 3 steps):

$$-\operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \operatorname{Log}[\operatorname{Sinh}[x]]$$

Result (type 3, 20 leaves):

$$-\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] + \operatorname{Log}[\operatorname{Sinh}[x]]$$

**Problem 674: Result more than twice size of optimal antiderivative.**

$$\int (\operatorname{Csch}[x] + \operatorname{Sinh}[x]) \, dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \operatorname{Cosh}[x]$$

Result (type 3, 19 leaves):

$$\operatorname{Cosh}[x] - \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right]$$

**Problem 677: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Csch}[x] + \operatorname{Sinh}[x]} \, dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$2 \sqrt{\operatorname{Cosh}[x] \operatorname{Coth}[x]} \operatorname{Tanh}[x]$$

Result (type 3, 35 leaves):

$$\frac{2 \sqrt{\operatorname{Cosh}[x] \operatorname{Coth}[x]} \left(-1 + (-\operatorname{Sinh}[x]^2)^{1/4}\right) \operatorname{Tanh}[x]}{(-\operatorname{Sinh}[x]^2)^{1/4}}$$



### Problem 687: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\text{Sinh}[x] - \text{Tanh}[x]} dx$$

Optimal (type 3, 20 leaves, 6 steps):

$$-\frac{1}{2} \text{ArcTanh}[\text{Cosh}[x]] + \frac{1}{2(1 - \text{Cosh}[x])}$$

Result (type 3, 50 leaves):

$$-\frac{1}{4} \text{Csch}\left[\frac{x}{2}\right]^2 \left(1 - \text{Log}[\text{Cosh}\left[\frac{x}{2}\right]] + \text{Cosh}[x] \left(\text{Log}[\text{Cosh}\left[\frac{x}{2}\right]] - \text{Log}[\text{Sinh}\left[\frac{x}{2}\right]]\right) + \text{Log}[\text{Sinh}\left[\frac{x}{2}\right]]\right)$$

### Problem 702: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[x]}{(a \text{Cosh}[x] + b \text{Sinh}[x])^3} dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$\frac{\text{Tanh}[x]^2}{2a(a + b \text{Tanh}[x])^2}$$

Result (type 3, 54 leaves):

$$-\frac{a^2 - b^2 + b^2 \text{Cosh}[2x] + a b \text{Sinh}[2x]}{2a(a - b)(a + b)(a \text{Cosh}[x] + b \text{Sinh}[x])^2}$$

### Problem 704: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cosh}[x]}{(a \text{Cosh}[x] + b \text{Sinh}[x])^3} dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$-\frac{\text{Coth}[x]^2}{2b(b + a \text{Coth}[x])^2}$$

Result (type 3, 40 leaves):

$$\frac{b \text{Cosh}[2x] + a \text{Sinh}[2x]}{2(a - b)(a + b)(a \text{Cosh}[x] + b \text{Sinh}[x])^2}$$

### Problem 745: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^4} dx$$

Optimal (type 3, 220 leaves, 6 steps):

$$\frac{a (2 a^2 + 3 b^2 - 3 c^2) \operatorname{ArcTanh}\left[\frac{c - (a-b) \operatorname{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^2 - b^2 + c^2}}\right]}{(a^2 - b^2 + c^2)^{7/2}} - \frac{c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{3 (a^2 - b^2 + c^2) (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^3} - \frac{5 (a c \operatorname{Cosh}[x] + a b \operatorname{Sinh}[x])}{6 (a^2 - b^2 + c^2)^2 (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^2} - \frac{c (11 a^2 + 4 b^2 - 4 c^2) \operatorname{Cosh}[x] + b (11 a^2 + 4 b^2 - 4 c^2) \operatorname{Sinh}[x]}{6 (a^2 - b^2 + c^2)^3 (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])}$$

Result (type 3, 488 leaves):

$$\frac{a (2 a^2 + 3 b^2 - 3 c^2) \operatorname{ArcTan}\left[\frac{c + (-a+b) \operatorname{Tanh}\left[\frac{x}{2}\right]}{\sqrt{-a^2 + b^2 - c^2}}\right]}{(-a^2 + b^2 - c^2)^{7/2}} - \frac{1}{24 b (a^2 - b^2 + c^2)^3 (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^3} (-44 a^5 c - 82 a^3 b^2 c - 24 a b^4 c + 82 a^3 c^3 + 48 a b^2 c^3 - 24 a c^5 - 30 a^2 b c (2 a^2 + 3 b^2 - 3 c^2) \operatorname{Cosh}[x] - 6 a c (a^2 (-7 b^2 + 11 c^2) + 2 (b^4 + b^2 c^2 - 2 c^4)) \operatorname{Cosh}[2 x] + 22 a^2 b^3 c \operatorname{Cosh}[3 x] + 8 b^5 c \operatorname{Cosh}[3 x] - 22 a^2 b c^3 \operatorname{Cosh}[3 x] - 16 b^3 c^3 \operatorname{Cosh}[3 x] + 8 b c^5 \operatorname{Cosh}[3 x] + 72 a^4 b^2 \operatorname{Sinh}[x] - 9 a^2 b^4 \operatorname{Sinh}[x] + 12 b^6 \operatorname{Sinh}[x] - 132 a^4 c^2 \operatorname{Sinh}[x] - 72 a^2 b^2 c^2 \operatorname{Sinh}[x] - 36 b^4 c^2 \operatorname{Sinh}[x] + 81 a^2 c^4 \operatorname{Sinh}[x] + 36 b^2 c^4 \operatorname{Sinh}[x] - 12 c^6 \operatorname{Sinh}[x] + 54 a^3 b^3 \operatorname{Sinh}[2 x] + 6 a b^5 \operatorname{Sinh}[2 x] - 78 a^3 b c^2 \operatorname{Sinh}[2 x] - 48 a b^3 c^2 \operatorname{Sinh}[2 x] + 42 a b c^4 \operatorname{Sinh}[2 x] + 11 a^2 b^4 \operatorname{Sinh}[3 x] + 4 b^6 \operatorname{Sinh}[3 x] - 4 b^4 c^2 \operatorname{Sinh}[3 x] - 11 a^2 c^4 \operatorname{Sinh}[3 x] - 4 b^2 c^4 \operatorname{Sinh}[3 x] + 4 c^6 \operatorname{Sinh}[3 x])$$

### Problem 749: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{a + a \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\operatorname{Log}\left[a + c \operatorname{Tanh}\left[\frac{x}{2}\right]\right]}{c}$$

Result (type 3, 35 leaves):

$$\frac{\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right]}{c} + \frac{\operatorname{Log}\left[a \operatorname{Cosh}\left[\frac{x}{2}\right] + c \operatorname{Sinh}\left[\frac{x}{2}\right]\right]}{c}$$

### Problem 750: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^2} dx$$

Optimal (type 3, 43 leaves, 4 steps):

$$\frac{a \operatorname{Log}\left[a + c \operatorname{Tanh}\left[\frac{x}{2}\right]\right]}{c^3} - \frac{c \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]}{c^2 (a + a \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])}$$

Result (type 3, 87 leaves):

$$\frac{2 a \left( -\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[a \operatorname{Cosh}\left[\frac{x}{2}\right] + c \operatorname{Sinh}\left[\frac{x}{2}\right]\right] \right) + \frac{c (-a^2 + c^2) \operatorname{Sinh}\left[\frac{x}{2}\right]}{a (a \operatorname{Cosh}\left[\frac{x}{2}\right] + c \operatorname{Sinh}\left[\frac{x}{2}\right])} - c \operatorname{Tanh}\left[\frac{x}{2}\right]}{2 c^3}$$

### Problem 752: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^4} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$\frac{a (5 a^2 - 3 c^2) \operatorname{Log}\left[a + c \operatorname{Tanh}\left[\frac{x}{2}\right]\right]}{2 c^7} - \frac{c \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]}{3 c^2 (a + a \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^3} - \frac{5 (a c \operatorname{Cosh}[x] + a^2 \operatorname{Sinh}[x])}{6 c^4 (a + a \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^2} - \frac{c (15 a^2 - 4 c^2) \operatorname{Cosh}[x] + a (15 a^2 - 4 c^2) \operatorname{Sinh}[x]}{6 c^6 (a + a \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])}$$

Result (type 3, 300 leaves):

$$\frac{1}{384 c^7} \left( 192 (-5 a^3 + 3 a c^2) \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + 192 a (5 a^2 - 3 c^2) \operatorname{Log}\left[a \operatorname{Cosh}\left[\frac{x}{2}\right] + c \operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \frac{1}{a \left(a + c \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^3} c \operatorname{Sech}\left[\frac{x}{2}\right]^6 \left( -150 a^5 c + 130 a^3 c^3 - 24 a c^5 + (-75 a^5 c + 75 a^3 c^3 + 12 a c^5) \operatorname{Cosh}[x] + 6 a c (25 a^4 - 15 a^2 c^2 + 4 c^4) \operatorname{Cosh}[2 x] + 75 a^5 c \operatorname{Cosh}[3 x] - 35 a^3 c^3 \operatorname{Cosh}[3 x] + 4 a c^5 \operatorname{Cosh}[3 x] + 150 a^6 \operatorname{Sinh}[x] - 255 a^4 c^2 \operatorname{Sinh}[x] + 129 a^2 c^4 \operatorname{Sinh}[x] - 12 c^6 \operatorname{Sinh}[x] + 120 a^6 \operatorname{Sinh}[2 x] - 72 a^4 c^2 \operatorname{Sinh}[2 x] + 36 a^2 c^4 \operatorname{Sinh}[2 x] + 30 a^6 \operatorname{Sinh}[3 x] + 37 a^4 c^2 \operatorname{Sinh}[3 x] - 27 a^2 c^4 \operatorname{Sinh}[3 x] + 4 c^6 \operatorname{Sinh}[3 x] \right) \right)$$

**Problem 760: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^4} dx$$

Optimal (type 3, 198 leaves, 4 steps):

$$\frac{c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{7 \sqrt{b^2 - c^2} \left(\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^4} + \frac{3 \left(c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]\right)}{35 \left(b^2 - c^2\right) \left(\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^3} +$$

$$\frac{2 \left(c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]\right)}{35 \left(b^2 - c^2\right)^{3/2} \left(\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^2} - \frac{2 \left(c + \sqrt{b^2 - c^2} \operatorname{Sinh}[x]\right)}{35 c \left(b^2 - c^2\right)^{3/2} \left(c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]\right)}$$

Result (type 3, 425 leaves):

$$-\frac{1}{1120 (b - c) c (b + c) (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])^7}$$

$$\left( -832 b^4 c \sqrt{b^2 - c^2} + 1664 b^2 c^3 \sqrt{b^2 - c^2} - 832 c^5 \sqrt{b^2 - c^2} + 1190 b c (b^2 - c^2)^2 \operatorname{Cosh}[x] + 448 c \sqrt{b^2 - c^2} (-b^4 + c^4) \operatorname{Cosh}[2x] + \right.$$

$$112 b^5 c \operatorname{Cosh}[3x] + 56 b^3 c^3 \operatorname{Cosh}[3x] - 168 b c^5 \operatorname{Cosh}[3x] - 28 b^5 c \operatorname{Cosh}[5x] + 28 b c^5 \operatorname{Cosh}[5x] + 6 b^5 c \operatorname{Cosh}[7x] + 20 b^3 c^3 \operatorname{Cosh}[7x] +$$

$$6 b c^5 \operatorname{Cosh}[7x] - 35 b^6 \operatorname{Sinh}[x] + 1295 b^4 c^2 \operatorname{Sinh}[x] - 2485 b^2 c^4 \operatorname{Sinh}[x] + 1225 c^6 \operatorname{Sinh}[x] - 896 b^3 c^2 \sqrt{b^2 - c^2} \operatorname{Sinh}[2x] +$$

$$896 b c^4 \sqrt{b^2 - c^2} \operatorname{Sinh}[2x] + 21 b^6 \operatorname{Sinh}[3x] + 189 b^4 c^2 \operatorname{Sinh}[3x] - 161 b^2 c^4 \operatorname{Sinh}[3x] - 49 c^6 \operatorname{Sinh}[3x] - 7 b^6 \operatorname{Sinh}[5x] -$$

$$\left. 35 b^4 c^2 \operatorname{Sinh}[5x] + 35 b^2 c^4 \operatorname{Sinh}[5x] + 7 c^6 \operatorname{Sinh}[5x] + b^6 \operatorname{Sinh}[7x] + 15 b^4 c^2 \operatorname{Sinh}[7x] + 15 b^2 c^4 \operatorname{Sinh}[7x] + c^6 \operatorname{Sinh}[7x] \right)$$

**Problem 761: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{5/2} dx$$

Optimal (type 4, 294 leaves, 7 steps):

$$\frac{16}{15} (a c \operatorname{Cosh}[x] + a b \operatorname{Sinh}[x]) \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} + \frac{2}{5} (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]) (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{3/2} -$$

$$\frac{2 i (23 a^2 + 9 b^2 - 9 c^2) \operatorname{EllipticE}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{15 \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}} +$$

$$\frac{16 i a (a^2 - b^2 + c^2) \operatorname{EllipticF}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}}{15 \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}$$

Result (type 6, 3775 leaves):

$$\sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left( \frac{2 b (23 a^2 + 9 b^2 - 9 c^2)}{15 c} + \frac{22}{15} a c \operatorname{Cosh}[x] + \frac{2}{5} b c \operatorname{Cosh}[2 x] + \frac{22}{15} a b \operatorname{Sinh}[x] + \frac{1}{5} (b^2 + c^2) \operatorname{Sinh}[2 x] \right) +$$

$$\left( 2 a^3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)$$

$$\sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} - i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} + i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}}$$

$$\sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \left/ \left( \sqrt{1 - \frac{b^2}{c^2}} c \sqrt{i \left( i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) +$$

$$\left( 34 a b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)$$

$$\sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} - i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} + i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}}$$

$$\sqrt{a + c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \left/ \left( 15 \sqrt{1 - \frac{b^2}{c^2}} c \sqrt{i \left( i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) - \right.$$

$$\left( 34 a c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)$$

$$\sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} - i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} + i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}}$$

$$\left( \sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \right) / \left( 15 \sqrt{1 - \frac{b^2}{c^2}} \sqrt{i \left( i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) - \frac{1}{15 c}$$

$$23 a^2 b^2 \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) / \right)$$

$$\left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} - b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2 - c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2 - c^2}{b^2}}} - \frac{2 b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right}}} \right) - \frac{1}{5 c}$$

$$3 b^4 \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) / \right)$$

$$\begin{aligned}
& \left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right}}} \right) + \\
& \frac{23}{15} a^2 c \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)} \right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) / \\
& \left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right}}} \right) +
\end{aligned}$$



$$\frac{6}{5} b^2 c \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a+b\sqrt{1-\frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1-\frac{c^2}{b^2}} \left( 1 + \frac{a}{b\sqrt{1-\frac{c^2}{b^2}}} \right)}, \frac{a+b\sqrt{1-\frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1-\frac{c^2}{b^2}} \left( -1 + \frac{a}{b\sqrt{1-\frac{c^2}{b^2}}} \right)} \right] \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right] \right) / \right.$$

$$\left( b\sqrt{1-\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}} - b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2-c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]} \right.$$

$$\left. \sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}} + b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2-c^2}{b^2}}} \frac{-\frac{2b\left(a+b\sqrt{1-\frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]\right)}{b^2-c^2} + \frac{c \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1-\frac{c^2}{b^2}}}}{\sqrt{a+b\sqrt{1-\frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}} \right) -$$

$$\frac{3}{5} c^3 \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a+b\sqrt{1-\frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1-\frac{c^2}{b^2}} \left( 1 + \frac{a}{b\sqrt{1-\frac{c^2}{b^2}}} \right)}, \frac{a+b\sqrt{1-\frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1-\frac{c^2}{b^2}} \left( -1 + \frac{a}{b\sqrt{1-\frac{c^2}{b^2}}} \right)} \right] \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right] \right) / \right.$$

$$\left( b\sqrt{1-\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}} - b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2-c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]} \right.$$

$$\sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}}$$

**Problem 762: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{3/2} dx$$

Optimal (type 4, 249 leaves, 6 steps):

$$\frac{2}{3} (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]) \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} - \frac{8 i a \operatorname{EllipticE}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2-c^2}}{a + \sqrt{b^2-c^2}}\right] \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{3 \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2-c^2}}}}$$

$$\frac{2 i (a^2 - b^2 + c^2) \operatorname{EllipticF}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2-c^2}}{a + \sqrt{b^2-c^2}}\right] \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2-c^2}}}}{3 \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}$$

Result (type 6, 2292 leaves):

$$\left( \frac{8 a b}{3 c} + \frac{2}{3} c \operatorname{Cosh}[x] + \frac{2}{3} b \operatorname{Sinh}[x] \right) \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} +$$

$$\left[ 2 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)}{\sqrt{1 - \frac{b^2}{c^2}} \left( 1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c} \right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)}{\sqrt{1 - \frac{b^2}{c^2}} \left( -1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c} \right) c} \right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]$$

$$\begin{aligned}
& \sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} - i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} + i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \\
& \left. \sqrt{a + c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \right) / \left( \sqrt{1 - \frac{b^2}{c^2}} c \sqrt{i \left(i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) + \\
& \left( 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left(a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left(a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right) \\
& \sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} - i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} + i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \\
& \left. \sqrt{a + c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \right) / \left( 3 \sqrt{1 - \frac{b^2}{c^2}} c \sqrt{i \left(i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) -
\end{aligned}$$

$$\left( 2 c \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right) \right)}{\sqrt{1 - \frac{b^2}{c^2}} \left( 1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c} \right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right) \right)}{\sqrt{1 - \frac{b^2}{c^2}} \left( -1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c} \right) c} \right] \operatorname{Sech} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right] \right.$$

$$\left. \frac{\sqrt{-1 + i \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]} \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} - i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}}{\sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} + i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{-i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}} \right)$$

$$\left. \frac{\sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]} \right) \left/ \left( 3 \sqrt{1 - \frac{b^2}{c^2}} \sqrt{i \left( i + \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right) \right)} \right) - \frac{1}{3 c} \right.$$

$$\left. \left( 4 a b^2 \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right)}{b \sqrt{1 - \frac{c^2}{b^2}} \left( 1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}} \right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left( -1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}} \right)} \right] \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right] \right) \right/$$

$$\left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} - b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 - c^2}{b^2}}}} \sqrt{\frac{a + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b^2}} \right)$$

$$\begin{aligned}
& \left( \frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{b^2 - c^2} \right) \frac{1}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}} + \\
& \frac{4}{3} a c \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) \right) / \\
& \left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}} \right) \\
& \left( \frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{b^2 - c^2} \right) \frac{1}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}}
\end{aligned}$$

**Problem 763:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \, dx$$

Optimal (type 4, 102 leaves, 2 steps):

$$\frac{2 i \operatorname{EllipticE}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{\sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}}$$

Result (type 6, 1401 leaves):

$$\frac{2 b \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{c} +$$

$$\left( 2 a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)$$

$$\sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} - i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} + i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}}$$

$$\sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \left/ \left( \sqrt{1 - \frac{b^2}{c^2}} c \sqrt{i \left( i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) - \frac{1}{c} \right)$$

$$b^2 \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) \right/$$

$$\begin{aligned}
& \left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right}}} \right) + \\
& c \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)} \right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) / \right. \\
& \left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right}}} \right)
\end{aligned}$$

Problem 764: Result unnecessarily involves higher level functions and more than twice size of optimal

## antiderivative.

$$\int \frac{1}{\sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}} dx$$

Optimal (type 4, 102 leaves, 2 steps):

$$\frac{2 i \operatorname{EllipticF}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}}{\sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}$$

Result (type 6, 237 leaves):

$$\frac{1}{\sqrt{1 - \frac{b^2}{c^2} c}} {}_2\operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + i \sqrt{1 - \frac{b^2}{c^2} c}}, \frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a - i \sqrt{1 - \frac{b^2}{c^2} c}}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]$$

$$\frac{\sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{\sqrt{\frac{-i \sqrt{1 - \frac{b^2}{c^2} c} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + i \sqrt{1 - \frac{b^2}{c^2} c}} - \frac{i \sqrt{1 - \frac{b^2}{c^2} c} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a - i \sqrt{1 - \frac{b^2}{c^2} c}}}}$$

Problem 765: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{3/2}} dx$$

Optimal (type 4, 156 leaves, 3 steps):

$$\frac{2 (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{(a^2 - b^2 + c^2) \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}} - \frac{2 i \operatorname{EllipticE}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{(a^2 - b^2 + c^2) \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}}$$

Result (type 6, 1522 leaves):

$$\sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left( -\frac{2 (b^2 - c^2)}{b c (-a^2 + b^2 - c^2)} - \frac{2 (a c - b^2 \operatorname{Sinh}[x] + c^2 \operatorname{Sinh}[x])}{b (-a^2 + b^2 - c^2) (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])} \right) +$$



$$\left( 2 a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)$$

$$\sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} - i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} + i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}}$$

$$\sqrt{a + c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \left/ \left( \sqrt{1 - \frac{b^2}{c^2}} c (a^2 - b^2 + c^2) \sqrt{i \left(i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) - \frac{1}{c (a^2 - b^2 + c^2)} \right)$$

$$b^2 \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right/$$

$$\left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right)$$

$$\begin{aligned}
& \left( \frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{b^2 - c^2}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}} \right) + \frac{1}{a^2 - b^2 + c^2} \\
& c \left( \left( \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) \right) / \\
& \left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right) \\
& \left( \frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{b^2 - c^2}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}} \right)
\end{aligned}$$

**Problem 766:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{5/2}} dx$$

Optimal (type 4, 322 leaves, 7 steps):

$$\frac{2 (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{3 (a^2 - b^2 + c^2) (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{3/2}} - \frac{8 (a c \operatorname{Cosh}[x] + a b \operatorname{Sinh}[x])}{3 (a^2 - b^2 + c^2)^2 \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}$$

$$\frac{8 i a \operatorname{EllipticE}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{3 (a^2 - b^2 + c^2)^2 \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}} +$$

$$\frac{2 i \operatorname{EllipticF}\left[\frac{1}{2} (i x - \operatorname{ArcTan}[b, -i c]), \frac{2 \sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}}{3 (a^2 - b^2 + c^2) \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}$$

Result (type 6, 2492 leaves):

$$\sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left( \frac{8 a (b^2 - c^2)}{3 b c (a^2 - b^2 + c^2)^2} - \frac{2 (a c - b^2 \operatorname{Sinh}[x] + c^2 \operatorname{Sinh}[x])}{3 b (-a^2 + b^2 - c^2) (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^2} - \frac{2 (-3 a^2 c - b^2 c + c^3 + 4 a b^2 \operatorname{Sinh}[x] - 4 a c^2 \operatorname{Sinh}[x])}{3 b (-a^2 + b^2 - c^2)^2 (a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])} \right) +$$

$$\left( 2 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right.$$

$$\left. \sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} - i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} + i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}} \right.$$

$$\left. \sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \right) / \left( \sqrt{1 - \frac{b^2}{c^2}} c (a^2 - b^2 + c^2)^2 \sqrt{i \left( i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) +$$

$$\left( 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)$$

$$\sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} - i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} + i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}}$$

$$\sqrt{a + c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \left/ \left( 3 \sqrt{1 - \frac{b^2}{c^2}} c (a^2 - b^2 + c^2)^2 \sqrt{i \left( i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) - \right.$$

$$\left( 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \right)$$

$$\sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} - i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}} \sqrt{\frac{c \sqrt{\frac{-b^2+c^2}{c^2}} + i c \sqrt{\frac{-b^2+c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2+c^2}{c^2}}}}$$

$$\sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \left/ \left( 3 \sqrt{1 - \frac{b^2}{c^2}} (a^2 - b^2 + c^2)^2 \sqrt{i \left( i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) - \frac{1}{3 c (a^2 - b^2 + c^2)^2} \right.$$

$$4 a b^2 \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) \right/$$

$$\left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} - b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2 - c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2 - c^2}{b^2}}} - \frac{2 b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right}}} \right) + \frac{1}{3 (a^2 - b^2 + c^2)^2}$$

$$4 a c \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) \right/$$

$$\left( \frac{b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} - b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2 - c^2}{b^2}}}}{\sqrt{a + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}} \right. \\ \left. - \frac{\sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2 - c^2}{b^2}}}}{-\frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right)}{b^2 - c^2} + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}} \right)$$

**Problem 767:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{7/2}} dx$$

Optimal (type 4, 411 leaves, 8 steps):

$$\frac{2(c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{5(a^2 - b^2 + c^2)(a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{5/2}} - \frac{16(ac \operatorname{Cosh}[x] + ab \operatorname{Sinh}[x])}{15(a^2 - b^2 + c^2)^2(a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{3/2}} - \\ \frac{2i(23a^2 + 9b^2 - 9c^2) \operatorname{EllipticE}\left[\frac{1}{2}(ix - \operatorname{ArcTan}[b, -ic]), \frac{2\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{15(a^2 - b^2 + c^2)^3 \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}} + \\ \frac{16ia \operatorname{EllipticF}\left[\frac{1}{2}(ix - \operatorname{ArcTan}[b, -ic]), \frac{2\sqrt{b^2 - c^2}}{a + \sqrt{b^2 - c^2}}\right] \sqrt{\frac{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{a + \sqrt{b^2 - c^2}}}}{15(a^2 - b^2 + c^2)^2 \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}} - \frac{2(c(23a^2 + 9b^2 - 9c^2) \operatorname{Cosh}[x] + b(23a^2 + 9b^2 - 9c^2) \operatorname{Sinh}[x])}{15(a^2 - b^2 + c^2)^3 \sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}$$

Result (type 6, 4093 leaves):

$$\sqrt{a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \\ \left( -\frac{2(23a^2 + 9b^2 - 9c^2)(b^2 - c^2)}{15bc(-a^2 + b^2 - c^2)^3} - \frac{2(ac - b^2 \operatorname{Sinh}[x] + c^2 \operatorname{Sinh}[x])}{5b(-a^2 + b^2 - c^2)(a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^3} - \frac{2(-5a^2c - 3b^2c + 3c^3 + 8ab^2 \operatorname{Sinh}[x] - 8ac^2 \operatorname{Sinh}[x])}{15b(-a^2 + b^2 - c^2)^2(a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^2} \right) +$$

$$\left( 2 \left( -15 a^3 c - 17 a b^2 c + 17 a c^3 + 23 a^2 b^2 \operatorname{Sinh}[x] + 9 b^4 \operatorname{Sinh}[x] - 23 a^2 c^2 \operatorname{Sinh}[x] - 18 b^2 c^2 \operatorname{Sinh}[x] + 9 c^4 \operatorname{Sinh}[x] \right) \right) /$$

$$\left( 15 b \left( -a^2 + b^2 - c^2 \right)^3 \left( a + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x] \right) \right) +$$

$$\left( 2 a^3 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right] \right)}{\sqrt{1 - \frac{b^2}{c^2}} \left( 1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c} \right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right] \right)}{\sqrt{1 - \frac{b^2}{c^2}} \left( -1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c} \right) c} \right] \operatorname{Sech} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right] \right.$$

$$\frac{\sqrt{-1 + i \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{\sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} - i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}}}{\sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} + i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{-i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}}$$

$$\left. \frac{\sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{\left( \sqrt{1 - \frac{b^2}{c^2}} c \left( a^2 - b^2 + c^2 \right)^3 \sqrt{i \left( i + \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right] \right)} \right)} \right) +$$

$$\left( 34 a b^2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right] \right)}{\sqrt{1 - \frac{b^2}{c^2}} \left( 1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c} \right) c}, -\frac{i \left( a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right] \right)}{\sqrt{1 - \frac{b^2}{c^2}} \left( -1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c} \right) c} \right] \operatorname{Sech} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right] \right.$$

$$\frac{\sqrt{-1 + i \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{\sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} - i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}}}{\sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} + i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{b}{c} \right] \right]}{-i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}}$$

$$\left. \sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \right) / \left( 15 \sqrt{1 - \frac{b^2}{c^2}} c (a^2 - b^2 + c^2)^3 \sqrt{i \left(i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) -$$

$$\left( 34 a c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{i \left(a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}, -\frac{i \left(a + \sqrt{1 - \frac{b^2}{c^2}} c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)}{\sqrt{1 - \frac{b^2}{c^2}} \left(-1 - \frac{i a}{\sqrt{1 - \frac{b^2}{c^2}} c}\right) c}\right] \right)$$

$$\operatorname{Sech}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right] \sqrt{-1 + i \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} - i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}}$$

$$\left. \sqrt{\frac{c \sqrt{\frac{-b^2 + c^2}{c^2}} + i c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]}{-i a + c \sqrt{\frac{-b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{-b^2 + c^2}{c^2}} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]} \right) /$$

$$\left( 15 \sqrt{1 - \frac{b^2}{c^2}} (a^2 - b^2 + c^2)^3 \sqrt{i \left(i + \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{b}{c}\right]\right]\right)} \right) - \frac{1}{15 c (a^2 - b^2 + c^2)^3}$$



$$23 a^2 b^2 \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left( 1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}} \right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left( -1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}} \right)} \right] \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} - b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 - c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \frac{b \sqrt{\frac{b^2 - c^2}{b^2}} + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 - c^2}{b^2}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right] \right) + \frac{c \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{b^2 - c^2}}{\sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}} - \frac{1}{5c (a^2 - b^2 + c^2)^3} \right)$$

$$3 b^4 \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left( 1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}} \right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left( -1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}} \right)} \right] \operatorname{Sinh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 - c^2}{b^2}} - b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 - c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2 - c^2}{b^2}} \operatorname{Cosh} \left[ x + \operatorname{ArcTanh} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}} + b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2-c^2}{b^2}}}}}{\sqrt{a + b\sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}} - \frac{2b\left(a + b\sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]\right)}{b^2 - c^2} + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 - \frac{c^2}{b^2}}} \right) + \frac{1}{15(a^2 - b^2 + c^2)^3}$$

$$23 a^2 c \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b\sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b\sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b\sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b\sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) / \right.$$

$$\left( b\sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}} - b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b\sqrt{\frac{b^2-c^2}{b^2}}}} \sqrt{a + b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right)$$

$$\left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}} + b\sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2-c^2}{b^2}}}}}{\sqrt{a + b\sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}} - \frac{2b\left(a + b\sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]\right)}{b^2 - c^2} + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 - \frac{c^2}{b^2}}} \right) + \frac{1}{5(a^2 - b^2 + c^2)^3}$$

$$6 b^2 c \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b\sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b\sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b\sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b\sqrt{1 - \frac{c^2}{b^2}}}\right)}\right] \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) / \right.$$

$$\left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right.$$

$$\left. \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{b^2 - c^2}} \sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right) - \frac{1}{5 (a^2 - b^2 + c^2)^3}$$

$$3 c^3 \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)}, \frac{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}} \left(-1 + \frac{a}{b \sqrt{1 - \frac{c^2}{b^2}}}\right)} \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) \right) /$$

$$\left( b \sqrt{1 - \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} - b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2-c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right.$$

$$\left. \sqrt{\frac{b \sqrt{\frac{b^2-c^2}{b^2}} + b \sqrt{\frac{b^2-c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2-c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right] \right) + \frac{c \operatorname{Sinh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 - \frac{c^2}{b^2}}}}{b^2 - c^2}} \sqrt{a + b \sqrt{1 - \frac{c^2}{b^2}} \operatorname{Cosh}\left[x + \operatorname{ArcTanh}\left[\frac{c}{b}\right]\right]} \right)$$

Problem 768: Result unnecessarily involves higher level functions and more than twice size of optimal

## antiderivative.

$$\int \left( \sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x] \right)^{5/2} dx$$

Optimal (type 3, 140 leaves, 3 steps):

$$\frac{64 (b^2 - c^2) (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{15 \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}} + \frac{16}{15} \sqrt{b^2 - c^2} (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]) \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} + \frac{2}{5} (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]) \left( \sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x] \right)^{3/2}$$

Result (type 4, 10223 leaves):

$$\begin{aligned} & \sqrt{b^2 - c^2} \left( \frac{4 b \sqrt{b^2 - c^2}}{3 c} + \frac{4}{3} c \operatorname{Cosh}[x] + \frac{4}{3} b \operatorname{Sinh}[x] \right) \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} + \\ & \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left( \frac{44 b (b^2 - c^2)}{15 c} + \frac{2}{15} c \sqrt{b^2 - c^2} \operatorname{Cosh}[x] + \frac{2}{5} b c \operatorname{Cosh}[2x] + \frac{2}{15} b \sqrt{b^2 - c^2} \operatorname{Sinh}[x] + \frac{1}{5} (b^2 + c^2) \operatorname{Sinh}[2x] \right) + \\ & \left( 256 b (-b + c) (b + c)^2 \sqrt{b^2 - c^2} \right. \\ & \left. \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2 \operatorname{EllipticPi} \left[ -1, \operatorname{ArcSin} \left[ \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] \right) \right. \\ & \left. \sqrt{\sqrt{(b - c) (b + c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \left( -c + (-b + \sqrt{b^2 - c^2}) \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \right) / \right. \\ & \left. \left( 15 (b + c - \sqrt{b^2 - c^2})^2 (b + c + \sqrt{b^2 - c^2}) (1 + \operatorname{Cosh}[x]) \sqrt{\frac{\sqrt{(b - c) (b + c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{(1 + \operatorname{Cosh}[x])^2}} \right. \right. \\ & \left. \left. \sqrt{\left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( -2 c \operatorname{Tanh} \left[ \frac{x}{2} \right] + \sqrt{b^2 - c^2} \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 - b \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 \right)} \right) \right) - \end{aligned}$$

$$\begin{aligned}
& \frac{1}{15 c (1 + \text{Cosh}[x]) \sqrt{\frac{\sqrt{(b-c)(b+c)} + b \text{Cosh}[x] + c \text{Sinh}[x]}{(1 + \text{Cosh}[x])^2}}} 64 (b-c)^2 (b+c)^2 \sqrt{\sqrt{(b-c)(b+c)} + b \text{Cosh}[x] + c \text{Sinh}[x]} \\
& \left( \left( (b-c \text{Tanh}\left[\frac{x}{2}\right]) \sqrt{-b - \sqrt{b^2 - c^2} - 2c \text{Tanh}\left[\frac{x}{2}\right] - b \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{b^2 - c^2} \text{Tanh}\left[\frac{x}{2}\right]^2} \right. \right. \\
& \left. \left. \sqrt{\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right) / \right. \\
& \left. \left( (-b^2 + c^2) \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{-2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) + \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-b - \sqrt{(b-c)(b+c)} - 2c \text{Tanh}\left[\frac{x}{2}\right] - b \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{(b-c)(b+c)} \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right. \\
& \left. \sqrt{\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right. \\
& \left. \left( 2c^2 \left(-1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left( -\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \text{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \text{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] + 2 \right. \right. \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}{-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}, \text{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \text{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \text{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] \right) \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]\right) \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left(-\frac{c}{-b+\sqrt{b^2-c^2}}+\operatorname{Tanh}[\frac{x}{2}]\right)} \Bigg/ \left(\left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right)\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)\right) \\
& \sqrt{\left(\left(-1+\operatorname{Tanh}[\frac{x}{2}]^2\right)\left(-b-\sqrt{(b-c)(b+c)}-2c\operatorname{Tanh}[\frac{x}{2}]+\left(-b+\sqrt{(b-c)(b+c)}\right)\operatorname{Tanh}[\frac{x}{2}]^2\right)\right)} + \left(8b^3\left(-b+c+\sqrt{b^2-c^2}\right)\right) \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c\operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \left(-1+\operatorname{Tanh}[\frac{x}{2}]\right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \\
& \left(-\frac{c}{-b+\sqrt{b^2-c^2}}+\operatorname{Tanh}[\frac{x}{2}]\right) \Bigg/ \left(\left(b-c-\sqrt{b^2-c^2}\right)\left(-b-c+\sqrt{b^2-c^2}\right)\left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right)\left(-\frac{-b-\sqrt{b^2-c^2}}{c}+\frac{-b+\sqrt{b^2-c^2}}{c}\right)\right) \\
& \sqrt{\left(\left(-1+\operatorname{Tanh}[\frac{x}{2}]^2\right)\left(-b-\sqrt{(b-c)(b+c)}-2c\operatorname{Tanh}[\frac{x}{2}]+\left(-b+\sqrt{(b-c)(b+c)}\right)\operatorname{Tanh}[\frac{x}{2}]^2\right)\right)} - \\
& \left(4b^5\left(-b+c+\sqrt{b^2-c^2}\right)\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c\right. \\
& \left.\operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right]\right)
\end{aligned}$$

$$\begin{aligned}
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \\
& \left( c^2 (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \\
& \left( 4bc^2 \left( -b+c+\sqrt{b^2-c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[ \frac{(b+c-\sqrt{b^2-c^2}) \left( 1 + \frac{c}{-b+\sqrt{b^2-c^2}} \right)}{(b-c-\sqrt{b^2-c^2}) \left( -1 + \frac{c}{-b+\sqrt{b^2-c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] \right) \\
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \\
& \left( (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \\
& \left( 8b^2\sqrt{b^2-c^2} \left( -b+c+\sqrt{b^2-c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[ \frac{(b+c-\sqrt{b^2-c^2}) \left( 1 + \frac{c}{-b+\sqrt{b^2-c^2}} \right)}{(b-c-\sqrt{b^2-c^2}) \left( -1 + \frac{c}{-b+\sqrt{b^2-c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \\
& \left( (b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 8b^4\sqrt{b^2-c^2} \left( (-b+c+\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right) \\
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \\
& \left( c^2 (b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 4b(b^2-c^2) \left( (-b+c+\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left( (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{-b+\sqrt{b^2-c^2}}\right) \left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2\right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2)} \right) \right) - \\
& \left( 4b^3(b^2-c^2) \left( (-b+c+\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] \right) \right) \\
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left( c^2 (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{-b+\sqrt{b^2-c^2}}\right) \left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2\right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2)} \right) \right) + \\
& \left( 8b^3 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] \right) \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \Bigg/ \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \sqrt{\left((-1+\operatorname{Tanh}[\frac{x}{2}]^2\right) \left(-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}[\frac{x}{2}] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]^2\right)} \right) - \\
& \left( 4b^5 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \right) \right) \\
& \left. (-1+\operatorname{Tanh}[\frac{x}{2}]) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right) \Bigg/ \\
& \left( c^2 (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \right. \\
& \left. \sqrt{\left((-1+\operatorname{Tanh}[\frac{x}{2}]^2\right) \left(-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}[\frac{x}{2}] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]^2\right)} \right) - \\
& \left( 4bc^2 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \right) \right) (-1+\operatorname{Tanh}[\frac{x}{2}])
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right/ \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \sqrt{\left((-1+\operatorname{Tanh}[\frac{x}{2}])^2\right) \left(-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}[\frac{x}{2}] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]\right)} \right) - \\
& \left( 4b(b^2-c^2) \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \left(-1+\operatorname{Tanh}[\frac{x}{2}]\right) \right) \\
& \left. \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right/ \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \sqrt{\left((-1+\operatorname{Tanh}[\frac{x}{2}])^2\right) \left(-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}[\frac{x}{2}] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]\right)} \right) + \\
& \left( 4b^3(b^2-c^2) \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \right) \\
& \left(-1+\operatorname{Tanh}[\frac{x}{2}]\right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right/
\end{aligned}$$

$$\begin{aligned}
& \left( c^2 \left( -b - c - \sqrt{b^2 - c^2} \right) \left( b - c + \sqrt{b^2 - c^2} \right) \left( 1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( \frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left( -b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left( -b + \sqrt{(b-c)(b+c)} \right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \\
& \left( 2b^3 \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right], 1 \right] \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right] / \left( c \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left( -b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left( -b + \sqrt{(b-c)(b+c)} \right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 2bc \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right], 1 \right] \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right] / \left( \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left( -b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left( -b + \sqrt{(b-c)(b+c)} \right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 2b^2 \sqrt{b^2 - c^2} \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right], 1 \right] \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \bigg/ \left(c \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left( bc \left( 2 \left( \frac{1}{2} \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right) - \right. \right. \\
& \left. \frac{c \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right] + 2c \operatorname{EllipticPi}\left[\frac{1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}{-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right]}{(-b + \sqrt{b^2 - c^2}) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)} + \frac{(-b + \sqrt{b^2 - c^2}) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(-b + \sqrt{b^2 - c^2}) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)} \right) \\
& \left. \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) + \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \right. \right. \\
& \left. \left. \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \bigg/ \left(\sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \right. \\
& \left. \left. \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) + \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) \right) \right)
\end{aligned}$$



**Problem 769: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x] \right)^{3/2} dx$$

Optimal (type 3, 92 leaves, 2 steps):

$$\frac{8 \sqrt{b^2 - c^2} (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{3 \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}} + \frac{2}{3} (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]) \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}$$

Result (type 4, 10 141 leaves):

$$\begin{aligned} & \frac{2 b \sqrt{b^2 - c^2} \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{c} + \\ & \left( \frac{2 b \sqrt{b^2 - c^2}}{3 c} + \frac{2}{3} c \operatorname{Cosh}[x] + \frac{2}{3} b \operatorname{Sinh}[x] \right) \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} + \left( 32 b (-b + c) (b + c)^2 \right. \\ & \left. \left( \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2 \operatorname{EllipticPi}\left[ -1, \operatorname{ArcSin}\left[ \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] \right) \\ & \left. \sqrt{\sqrt{(b - c) (b + c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \left( -c + (-b + \sqrt{b^2 - c^2}) \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\ & \left( 3 (b + c - \sqrt{b^2 - c^2})^2 (b + c + \sqrt{b^2 - c^2}) (1 + \operatorname{Cosh}[x]) \sqrt{\frac{\sqrt{(b - c) (b + c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{(1 + \operatorname{Cosh}[x])^2}} \right. \\ & \left. \sqrt{\left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left( -2 c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) - b \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \end{aligned}$$

$$\frac{1}{3c(1 + \text{Cosh}[x]) \sqrt{\frac{\sqrt{(b-c)(b+c) + b \text{Cosh}[x] + c \text{Sinh}[x]}{(1 + \text{Cosh}[x])^2}}}} 8(b-c)(b+c) \sqrt{b^2 - c^2} \sqrt{\sqrt{(b-c)(b+c) + b \text{Cosh}[x] + c \text{Sinh}[x]} + b \text{Cosh}[x] + c \text{Sinh}[x]}$$

$$\left( \left( (b - c \text{Tanh}\left[\frac{x}{2}\right]) \sqrt{-b - \sqrt{b^2 - c^2} - 2c \text{Tanh}\left[\frac{x}{2}\right] - b \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{b^2 - c^2} \text{Tanh}\left[\frac{x}{2}\right]^2} \right. \right. \\ \left. \left. \sqrt{\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right) / \right. \\ \left. \left( (-b^2 + c^2) \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{-2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) + \right. \\ \left. \sqrt{\left(\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-b - \sqrt{(b-c)(b+c)} - 2c \text{Tanh}\left[\frac{x}{2}\right] - b \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{(b-c)(b+c)} \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right. \\ \left. \sqrt{\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right) \\ \left( 2c^2 \left(-1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(-\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \text{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \text{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] + 2 \right. \right.$$

$$\left. \left. \text{EllipticPi}\left[\frac{1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}{-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}, \text{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \text{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \text{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] \right) \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]\right)$$



$$\begin{aligned}
& \sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}\left(-\frac{c}{-b+\sqrt{b^2-c^2}}+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}\left/\left(\left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right)\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)\right)\right. \\
& \sqrt{\left(\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\left(-b-\sqrt{(b-c)(b+c)}-2c\operatorname{Tanh}\left[\frac{x}{2}\right]+\left(-b+\sqrt{(b-c)(b+c)}\right)\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)}+8b^3\left(-b+c+\sqrt{b^2-c^2}\right) \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right],1\right]-2c\operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}\right], \\
& \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right],1\right)\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}} \\
& \left(-\frac{c}{-b+\sqrt{b^2-c^2}}+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)\left/\left(\left(b-c-\sqrt{b^2-c^2}\right)\left(-b-c+\sqrt{b^2-c^2}\right)\left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right)\left(-\frac{-b-\sqrt{b^2-c^2}}{c}+\frac{-b+\sqrt{b^2-c^2}}{c}\right)\right)\right. \\
& \left.\sqrt{\left(\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\left(-b-\sqrt{(b-c)(b+c)}-2c\operatorname{Tanh}\left[\frac{x}{2}\right]+\left(-b+\sqrt{(b-c)(b+c)}\right)\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)}\right)- \\
& \left(4b^5\left(-b+c+\sqrt{b^2-c^2}\right)\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right],1\right)-2c\right. \\
& \left.\operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}\right],\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right],1\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \Bigg) / \\
& \left( c^2 (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \\
& \left( 4bc^2 \left( -b+c+\sqrt{b^2-c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \\
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \Bigg) / \\
& \left( (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \\
& \left( 8b^2\sqrt{b^2-c^2} \left( -b+c+\sqrt{b^2-c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \Bigg/ \\
& \left( (b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 8b^4\sqrt{b^2-c^2} \left( (-b+c+\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right) \\
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \Bigg/ \\
& \left( c^2 (b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 4b(b^2-c^2) \left( (-b+c+\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( (b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \\
& \left( 4b^3(b^2-c^2) \left( (-b+c+\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right) \\
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( -\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( c^2(b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( 1 - \frac{c}{-b+\sqrt{b^2-c^2}} \right) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 8b^3 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right/ \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \sqrt{\left((-1+\operatorname{Tanh}[\frac{x}{2}]^2\right) \left(-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}[\frac{x}{2}] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]^2\right)} \right) - \\
& \left( 4b^5 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \right) \right) \\
& \left. (-1+\operatorname{Tanh}[\frac{x}{2}]) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right/ \\
& \left( c^2 (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \right. \\
& \left. \sqrt{\left((-1+\operatorname{Tanh}[\frac{x}{2}]^2\right) \left(-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}[\frac{x}{2}] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]^2\right)} \right) - \\
& \left( 4bc^2 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \right) \left(-1+\operatorname{Tanh}[\frac{x}{2}]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right/ \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \sqrt{\left((-1+\operatorname{Tanh}[\frac{x}{2}])^2\right) \left(-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}[\frac{x}{2}] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]\right)} \right) - \\
& \left( 4b(b^2-c^2) \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \left(-1+\operatorname{Tanh}[\frac{x}{2}]\right) \right) \\
& \left. \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right/ \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left(1-\frac{c}{-b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \sqrt{\left((-1+\operatorname{Tanh}[\frac{x}{2}])^2\right) \left(-b-\sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}[\frac{x}{2}] + (-b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]\right)} \right) + \\
& \left( 4b^3(b^2-c^2) \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \right) \\
& \left(-1+\operatorname{Tanh}[\frac{x}{2}]\right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(-b+c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(-\frac{c}{-b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}]\right)} \right/
\end{aligned}$$

$$\begin{aligned}
& \left( c^2 \left( -b - c - \sqrt{b^2 - c^2} \right) \left( b - c + \sqrt{b^2 - c^2} \right) \left( 1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( \frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c} \right) \right. \\
& \quad \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left( -b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left( -b + \sqrt{(b-c)(b+c)} \right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \\
& \left( 2b^3 \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right], 1 \right] \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right. \\
& \quad \left. \sqrt{\frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right] / \left( c \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right) \\
& \quad \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left( -b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left( -b + \sqrt{(b-c)(b+c)} \right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 2bc \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right], 1 \right] \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right. \\
& \quad \left. \sqrt{\frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right] / \left( \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right) \\
& \quad \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left( -b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left( -b + \sqrt{(b-c)(b+c)} \right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 2b^2 \sqrt{b^2 - c^2} \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right], 1 \right] \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & \sqrt{\frac{\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \left(c \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)\right) \\
 & \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(-b - \sqrt{(b - c)(b + c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b - c)(b + c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
 & \left( bc \left( 2 \left( \frac{1}{2} \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b + c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right) - \right. \right. \\
 & \left. \left. \frac{c \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b + c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right] + 2c \operatorname{EllipticPi}\left[\frac{1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}{-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b + c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right] \right)}{\left(-b + \sqrt{b^2 - c^2}\right) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)} + \frac{\left(-b + \sqrt{b^2 - c^2}\right) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{\left(-b + \sqrt{b^2 - c^2}\right) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)} \right) \\
 & \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b + c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) + \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \right.} \\
 & \left. \left. \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \Bigg/ \left(\sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(-b - \sqrt{(b - c)(b + c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b - c)(b + c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)}\right) -
 \end{aligned}$$



$$\left( c \sqrt{b^2 - c^2} \left( 2 \left( \frac{1}{2} \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] - \right. \right.$$

$$\left. \frac{c \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] + 2 c \text{EllipticPi} \left[ \frac{1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}{-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}, \text{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] \right)}{(-b + \sqrt{b^2 - c^2}) \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) + (-b + \sqrt{b^2 - c^2}) \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}$$

$$\left. \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right) + \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \right. \right.$$

$$\left. \left. \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 \right) \left/ \left( \sqrt{\left( \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( -b - \sqrt{(b - c)(b + c)} - 2 c \text{Tanh} \left[ \frac{x}{2} \right] + \left( -b + \sqrt{(b - c)(b + c)} \right) \text{Tanh} \left[ \frac{x}{2} \right]^2 \right)} \right) \right) \right/$$

$$\left( (b - c)(b + c) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 \sqrt{-b - \sqrt{(b - c)(b + c)} - 2 c \text{Tanh} \left[ \frac{x}{2} \right] - b \text{Tanh} \left[ \frac{x}{2} \right]^2 + \sqrt{(b - c)(b + c)} \text{Tanh} \left[ \frac{x}{2} \right]^2} \right.$$

$$\left. \left. \sqrt{-2 c \text{Tanh} \left[ \frac{x}{2} \right] + \sqrt{b^2 - c^2} \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 - b \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2} \right) \right)$$

**Problem 770: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \, dx$$

Optimal (type 3, 37 leaves, 1 step):

$$\frac{2 (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{\sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}$$

Result (type 4, 10054 leaves):

$$\frac{2 b \sqrt{\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{c} - \left( 8 b (b + c) \sqrt{b^2 - c^2} \right. \\ \left. \left( \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2 \operatorname{EllipticPi} \left[ -1, \operatorname{ArcSin} \left[ \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] \right) \\ \sqrt{\sqrt{(b - c)(b + c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{-\frac{(-b - c + \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \left( -c + (-b + \sqrt{b^2 - c^2}) \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \right) / \\ \left( (b + c - \sqrt{b^2 - c^2})^2 (b + c + \sqrt{b^2 - c^2}) (1 + \operatorname{Cosh}[x]) \sqrt{\frac{\sqrt{(b - c)(b + c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{(1 + \operatorname{Cosh}[x])^2}} \right. \\ \left. \sqrt{\left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( -2 c \operatorname{Tanh} \left[ \frac{x}{2} \right] + \sqrt{b^2 - c^2} \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 - b \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 \right)} \right) - \\ \frac{1}{c (1 + \operatorname{Cosh}[x])} \sqrt{\frac{\sqrt{(b - c)(b + c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{(1 + \operatorname{Cosh}[x])^2}} 2 (b - c) (b + c) \sqrt{\sqrt{(b - c)(b + c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}$$

$$\left( \left( b - c \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-b - \sqrt{b^2 - c^2} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] - b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{b^2 - c^2} \operatorname{Tanh}\left[\frac{x}{2}\right]^2} \right.$$

$$\left. \sqrt{\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right) /$$

$$\left( (-b^2 + c^2) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{-2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) +$$

$$\left( \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] - b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{(b-c)(b+c)} \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right.$$

$$\left. \sqrt{\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) - b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right)$$

$$\left( \left( 2c^2 \left(-1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left( -\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] + 2 \right. \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}{-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \right.$$

$$\left. \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \right) / \left( \left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \right)$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left(8b^3 \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b + c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(b-c - \sqrt{b^2 - c^2})\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b + c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right]\right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b + c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \Bigg/ \\
& \left(\left(b-c - \sqrt{b^2 - c^2}\right)\left(-b-c + \sqrt{b^2 - c^2}\right)\left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right)\left(-\frac{-b - \sqrt{b^2 - c^2}}{c} + \frac{-b + \sqrt{b^2 - c^2}}{c}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(4b^5 \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b + c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(b-c - \sqrt{b^2 - c^2})\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b + c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right]\right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b + c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \Bigg/ \\
& \left(c^2 \left(b-c - \sqrt{b^2 - c^2}\right)\left(-b-c + \sqrt{b^2 - c^2}\right)\left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right)\left(-\frac{-b - \sqrt{b^2 - c^2}}{c} + \frac{-b + \sqrt{b^2 - c^2}}{c}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(4bc^2 \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2})\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(b-c - \sqrt{b^2 - c^2})\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2})\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right]\right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2})\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(\left(b-c - \sqrt{b^2 - c^2}\right)\left(-b-c + \sqrt{b^2 - c^2}\right)\left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(-\frac{-b - \sqrt{b^2 - c^2}}{c} + \frac{-b + \sqrt{b^2 - c^2}}{c}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(8b^2\sqrt{b^2 - c^2} \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2})\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(b-c - \sqrt{b^2 - c^2})\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2})\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right]\right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2})\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(\left(b-c - \sqrt{b^2 - c^2}\right)\left(-b-c + \sqrt{b^2 - c^2}\right)\left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(-\frac{-b - \sqrt{b^2 - c^2}}{c} + \frac{-b + \sqrt{b^2 - c^2}}{c}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left(8b^4 \sqrt{b^2 - c^2} \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(b-c - \sqrt{b^2 - c^2})\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right]\right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(c^2 (b-c - \sqrt{b^2 - c^2}) (-b-c + \sqrt{b^2 - c^2}) \left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(-\frac{-b - \sqrt{b^2 - c^2}}{c} + \frac{-b + \sqrt{b^2 - c^2}}{c}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left(4b(b^2 - c^2) \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(b-c - \sqrt{b^2 - c^2})\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right]\right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left((b-c - \sqrt{b^2 - c^2}) (-b-c + \sqrt{b^2 - c^2}) \left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(-\frac{-b - \sqrt{b^2 - c^2}}{c} + \frac{-b + \sqrt{b^2 - c^2}}{c}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(4b^3(b^2 - c^2) \left( \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right]}, 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c - \sqrt{b^2 - c^2})\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(b-c - \sqrt{b^2 - c^2})\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right]}, 1\right] \right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(c^2(b-c - \sqrt{b^2 - c^2}) \left(-b - c + \sqrt{b^2 - c^2}\right) \left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right) \left(-\frac{-b - \sqrt{b^2 - c^2}}{c} + \frac{-b + \sqrt{b^2 - c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \right. \\
& \left. \left(8b^3 \left( \left(-b + c - \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right]}, 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c + \sqrt{b^2 - c^2})\left(1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{(b-c + \sqrt{b^2 - c^2})\left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right]}, 1\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \right. \\
& \left. \sqrt{\frac{(b+c - \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(-b+c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \left(\left(-b - c - \sqrt{b^2 - c^2}\right) \left(b - c + \sqrt{b^2 - c^2}\right) \left(1 - \frac{c}{-b + \sqrt{b^2 - c^2}}\right)\right) \\
& \left(\frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c}\right) \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} -
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b^5 \left( (-b + c - \sqrt{b^2 - c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] - 2 c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + c + \sqrt{b^2 - c^2}) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}{(b - c + \sqrt{b^2 - c^2}) \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] \right) \right. \\
& \quad \left. \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \right) \Bigg/ \\
& \quad \left( c^2 \left( -b - c - \sqrt{b^2 - c^2} \right) \left( b - c + \sqrt{b^2 - c^2} \right) \left( 1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( \frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c} \right) \right. \\
& \quad \left. \sqrt{\left( \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( -b - \sqrt{(b - c)(b + c)} - 2 c \operatorname{Tanh} \left[ \frac{x}{2} \right] + \left( -b + \sqrt{(b - c)(b + c)} \right) \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) \right) \right) - \\
& \quad \left( 4 b c^2 \left( (-b + c - \sqrt{b^2 - c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] - 2 c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + c + \sqrt{b^2 - c^2}) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}{(b - c + \sqrt{b^2 - c^2}) \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] \right) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \right. \\
& \quad \left. \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}{(-b + c + \sqrt{b^2 - c^2}) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)}} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \right) \Bigg/ \left( \left( -b - c - \sqrt{b^2 - c^2} \right) \left( b - c + \sqrt{b^2 - c^2} \right) \left( 1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right. \\
& \quad \left. \left( \frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c} \right) \sqrt{\left( \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( -b - \sqrt{(b - c)(b + c)} - 2 c \operatorname{Tanh} \left[ \frac{x}{2} \right] + \left( -b + \sqrt{(b - c)(b + c)} \right) \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) \right) \right) -
\end{aligned}$$



$$\begin{aligned}
& \left( 4 b (b^2 - c^2) \left( (-b + c - \sqrt{b^2 - c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2 c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + c + \sqrt{b^2 - c^2}) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}{(b - c + \sqrt{b^2 - c^2}) \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] \right) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \right. \\
& \quad \left. \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \right) \Bigg/ \left( (-b - c - \sqrt{b^2 - c^2}) (b - c + \sqrt{b^2 - c^2}) \left( 1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right. \\
& \quad \left. \left( \frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c} \right) \sqrt{\left( (-1 + \operatorname{Tanh}[\frac{x}{2}]^2) (-b - \sqrt{(b - c)(b + c)} - 2 c \operatorname{Tanh}[\frac{x}{2}] + (-b + \sqrt{(b - c)(b + c)}) \operatorname{Tanh}[\frac{x}{2}]^2) \right)} \right) \Bigg) + \\
& \left( 4 b^3 (b^2 - c^2) \left( (-b + c - \sqrt{b^2 - c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2 c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + c + \sqrt{b^2 - c^2}) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}{(b - c + \sqrt{b^2 - c^2}) \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] \right) \right. \\
& \quad \left. (-1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{\frac{(b + c - \sqrt{b^2 - c^2}) (1 + \operatorname{Tanh}[\frac{x}{2}])}{(-b + c + \sqrt{b^2 - c^2}) (-1 + \operatorname{Tanh}[\frac{x}{2}])}} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \right) \Bigg/ \\
& \left( c^2 (-b - c - \sqrt{b^2 - c^2}) (b - c + \sqrt{b^2 - c^2}) \left( 1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( \frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c} \right) \right. \\
& \quad \left. \sqrt{\left( (-1 + \operatorname{Tanh}[\frac{x}{2}]^2) (-b - \sqrt{(b - c)(b + c)} - 2 c \operatorname{Tanh}[\frac{x}{2}] + (-b + \sqrt{(b - c)(b + c)}) \operatorname{Tanh}[\frac{x}{2}]^2) \right)} \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left( 2 b^3 \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right], 1 \right] \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right) / \left( c \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( -b - \sqrt{(b-c)(b+c)} - 2c \text{Tanh} \left[ \frac{x}{2} \right] + \left( -b + \sqrt{(b-c)(b+c)} \right) \text{Tanh} \left[ \frac{x}{2} \right]^2 \right)} \right) \right) + \\
& \left( 2 b c \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right], 1 \right] \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right) / \left( \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( -b - \sqrt{(b-c)(b+c)} - 2c \text{Tanh} \left[ \frac{x}{2} \right] + \left( -b + \sqrt{(b-c)(b+c)} \right) \text{Tanh} \left[ \frac{x}{2} \right]^2 \right)} \right) \right) + \\
& \left( 2 b^2 \sqrt{b^2 - c^2} \left( -1 - \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right], 1 \right] \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \left( -\frac{c}{-b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right) / \left( c \left( -1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left( b c \left( 2 \left( \frac{1}{2} \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - \right. \right. \\
& \left. \left. \frac{c \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \operatorname{EllipticPi}\left[\frac{1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}{-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] \right)}{\left(-b + \sqrt{b^2 - c^2}\right) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)} + \frac{\left(-b + \sqrt{b^2 - c^2}\right) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)}{\left(-b + \sqrt{b^2 - c^2}\right) \left(-1 + \frac{c}{-b + \sqrt{b^2 - c^2}}\right)} \right) \\
& \left. \left. \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}} \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) + \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \left(-\frac{c}{-b + \sqrt{b^2 - c^2}} + \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right) \right) / \left(\sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(-b - \sqrt{(b-c)(b+c)} - 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(-b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)}\right) - \\
& \left( c \sqrt{b^2 - c^2} \left( 2 \left( \frac{1}{2} \left( 1 + \frac{c}{-b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - \right. \right.
\end{aligned}$$

$$\frac{c \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right]}{(-b+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)} + \frac{2c \operatorname{EllipticPi}\left[\frac{1+\frac{c}{-b+\sqrt{b^2-c^2}}}{-1+\frac{c}{-b+\sqrt{b^2-c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right]}{(-b+\sqrt{b^2-c^2})\left(-1+\frac{c}{-b+\sqrt{b^2-c^2}}\right)}$$

$$\left. \left. \begin{aligned} & \left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{\frac{(b+c-\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(-b+c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}} \left(-\frac{c}{-b+\sqrt{b^2-c^2}}+\operatorname{Tanh}\left[\frac{x}{2}\right]\right) + \left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right) \left(-\frac{c}{-b+\sqrt{b^2-c^2}}+\right. \\ & \left.\left.\left.\left.\left.\left.\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \right) \right) \left(\sqrt{\left(\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2\left(-b-\sqrt{(b-c)(b+c)}-2c\operatorname{Tanh}\left[\frac{x}{2}\right]+\left(-b+\sqrt{(b-c)(b+c)}\right)\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)}\right)} \right) \right) \right) \left. \right) /
 \end{aligned} \right.$$

$$\left( (b-c)(b+c)\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \sqrt{-b-\sqrt{(b-c)(b+c)}-2c\operatorname{Tanh}\left[\frac{x}{2}\right]-b\operatorname{Tanh}\left[\frac{x}{2}\right]^2+\sqrt{(b-c)(b+c)}\operatorname{Tanh}\left[\frac{x}{2}\right]^2} \right.$$

$$\left. \left. \left. \left. \left. \sqrt{-2c\operatorname{Tanh}\left[\frac{x}{2}\right]+\sqrt{b^2-c^2}\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2-b\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2} \right) \right) \right) \right) \right)$$

**Problem 771:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\operatorname{Cosh}[x]+c\operatorname{Sinh}[x]}} dx$$

Optimal (type 3, 99 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{(b^2-c^2)^{1/4} \operatorname{Sinh}[x+i \operatorname{ArcTan}[b,-i c]]}{\sqrt{2} \sqrt{\sqrt{b^2-c^2} + \sqrt{b^2-c^2} \operatorname{Cosh}[x+i \operatorname{ArcTan}[b,-i c]]}}\right]}{(b^2-c^2)^{1/4}}$$

Result (type 4, 211 leaves):

$$-\left(\left(\left(\sqrt{2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\sqrt{b^2-c^2}-b \operatorname{Cosh}[x]-c \operatorname{Sinh}[x]}}{\sqrt{b^2-c^2}}}\right], 1\right] \left(b^2-c^2+b \sqrt{b^2-c^2} \operatorname{Cosh}[x]+c \sqrt{b^2-c^2} \operatorname{Sinh}[x]\right)\right.\right.\right. \\ \left.\left.\left.\frac{\sqrt{-b^2+c^2+b \sqrt{b^2-c^2} \operatorname{Cosh}[x]+c \sqrt{b^2-c^2} \operatorname{Sinh}[x]}}{b^2-c^2}\right) / \left(\sqrt{b^2-c^2} (c \operatorname{Cosh}[x]+b \operatorname{Sinh}[x]) \sqrt{\sqrt{b^2-c^2}+b \operatorname{Cosh}[x]+c \operatorname{Sinh}[x]}\right)\right)\right)$$

Problem 772: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b \operatorname{Cosh}[x]+c \operatorname{Sinh}[x]\right)^{3/2}} dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{(b^2-c^2)^{1/4} \operatorname{Sinh}[x+i \operatorname{ArcTan}[b,-i c]]}{\sqrt{2} \sqrt{\sqrt{b^2-c^2} + \sqrt{b^2-c^2} \operatorname{Cosh}[x+i \operatorname{ArcTan}[b,-i c]]}}\right]}{2 \sqrt{2} (b^2-c^2)^{3/4}} + \frac{c \operatorname{Cosh}[x]+b \operatorname{Sinh}[x]}{2 \sqrt{b^2-c^2} \left(\sqrt{b^2-c^2}+b \operatorname{Cosh}[x]+c \operatorname{Sinh}[x]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

Problem 773: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b \operatorname{Cosh}[x]+c \operatorname{Sinh}[x]\right)^{5/2}} dx$$

Optimal (type 3, 205 leaves, 5 steps):

$$\frac{3 \operatorname{ArcTan}\left[\frac{(b^2-c^2)^{1/4} \operatorname{Sinh}[x+i \operatorname{ArcTan}[b,-i c]]}{\sqrt{2} \sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}} \operatorname{Cosh}[x+i \operatorname{ArcTan}[b,-i c]]}\right]}{16 \sqrt{2} (b^2-c^2)^{5/4}} + \frac{c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{4 \sqrt{b^2-c^2} (\sqrt{b^2-c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{5/2}} + \frac{3 (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{16 (b^2-c^2) (\sqrt{b^2-c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x])^{3/2}}$$

Result (type 1, 1 leaves):

???

**Problem 774: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( -\sqrt{b^2-c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x] \right)^{5/2} dx$$

Optimal (type 3, 146 leaves, 3 steps):

$$\frac{64 (b^2-c^2) (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{15 \sqrt{-\sqrt{b^2-c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}} - \frac{16}{15} \sqrt{b^2-c^2} (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]) \sqrt{-\sqrt{b^2-c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} + \frac{2}{5} (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]) \left( -\sqrt{b^2-c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x] \right)^{3/2}$$

Result (type 4, 9943 leaves):

$$\sqrt{b^2-c^2} \left( \frac{4 b \sqrt{b^2-c^2}}{3 c} - \frac{4}{3} c \operatorname{Cosh}[x] - \frac{4}{3} b \operatorname{Sinh}[x] \right) \sqrt{-\sqrt{b^2-c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} + \sqrt{-\sqrt{b^2-c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left( \frac{44 b (b^2-c^2)}{15 c} - \frac{2}{15} c \sqrt{b^2-c^2} \operatorname{Cosh}[x] + \frac{2}{5} b c \operatorname{Cosh}[2 x] - \frac{2}{15} b \sqrt{b^2-c^2} \operatorname{Sinh}[x] + \frac{1}{5} (b^2+c^2) \operatorname{Sinh}[2 x] \right) + \left( 256 b c (-b+c) (b+c) \sqrt{b^2-c^2} (-b^2+c^2) \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - \right.$$

$$\begin{aligned}
& 2 \operatorname{EllipticPi} \left[ -1, \operatorname{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1 \right] \sqrt{-\sqrt{(b-c)(b+c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \\
& \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \left( -\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \right)^{3/2} \left( c + (b+\sqrt{b^2-c^2}) \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) \Big/ \\
& \left( 15 (b+c+\sqrt{b^2-c^2})^3 (-b^2+c^2+b\sqrt{b^2-c^2}) (1+\operatorname{Cosh}[x]) \sqrt{\frac{-\sqrt{(b-c)(b+c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{(1+\operatorname{Cosh}[x])^2}} \right. \\
& \left. \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right] \right)^2 \sqrt{-\left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) \left( 2c \operatorname{Tanh} \left[ \frac{x}{2} \right] + \sqrt{b^2-c^2} \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) + b \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) \right)} \right) - \\
& \frac{1}{15c(1+\operatorname{Cosh}[x])} \sqrt{\frac{-\sqrt{(b-c)(b+c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{(1+\operatorname{Cosh}[x])^2}} \sqrt{-\sqrt{(b-c)(b+c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \\
& \left( \left( b - c \operatorname{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{b - \sqrt{b^2-c^2} + 2c \operatorname{Tanh} \left[ \frac{x}{2} \right] + b \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 + \sqrt{b^2-c^2} \operatorname{Tanh} \left[ \frac{x}{2} \right]^2} \right. \\
& \left. \sqrt{-\left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) \left( 2c \operatorname{Tanh} \left[ \frac{x}{2} \right] + \sqrt{b^2-c^2} \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) + b \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) \right)} \right) \Big/ \\
& \left( (-b^2+c^2) \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) \sqrt{2c \operatorname{Tanh} \left[ \frac{x}{2} \right] + \sqrt{b^2-c^2} \left( -1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right) + b \left( 1 + \operatorname{Tanh} \left[ \frac{x}{2} \right]^2 \right)} \right) -
\end{aligned}$$

$$\left( \sqrt{\left( (-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])^2 \right) \left( b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{(b-c)(b+c)} \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right)} \right)$$

$$\sqrt{-\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \left( 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) + b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \right)}$$

$$\left( 2c^2 \left(-1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left( -\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] + 2 \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{1 - \frac{c}{b + \sqrt{b^2 - c^2}}}{-1 - \frac{c}{b + \sqrt{b^2 - c^2}}}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)$$

$$\sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])} \left(\frac{c}{b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \left( \left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \right)$$

$$\sqrt{\left( (-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])^2 \right) \left( b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (b + \sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right)} + \left( 8b^3 \left( -b + c + \sqrt{b^2 - c^2} \right) \right)$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] - 2c \operatorname{EllipticPi}\left[\frac{(b+c - \sqrt{b^2 - c^2}) \left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right)}{(b-c - \sqrt{b^2 - c^2}) \left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right)}, \right.$$



$$\begin{aligned}
& \left. \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\text{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\text{Tanh}[\frac{x}{2}])}} \right], 1 \right] \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\text{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\text{Tanh}[\frac{x}{2}])}} \\
& \left( \frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right) \Bigg/ \left( (b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \right. \\
& \left. \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \sqrt{\left( (-1+\text{Tanh}[\frac{x}{2}])^2 \right) \left( b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh}[\frac{x}{2}] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh}[\frac{x}{2}]^2 \right)} \right) \Bigg) - \\
& \left( 4b^5 \left( (-b+c+\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\text{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\text{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2c \right. \right. \\
& \left. \left. \text{EllipticPi} \left[ \frac{(b+c-\sqrt{b^2-c^2}) \left( 1 - \frac{c}{b+\sqrt{b^2-c^2}} \right)}{(b-c-\sqrt{b^2-c^2}) \left( -1 - \frac{c}{b+\sqrt{b^2-c^2}} \right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\text{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\text{Tanh}[\frac{x}{2}])}} \right], 1 \right] \right) \right. \\
& \left. \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\text{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\text{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right) \right) \Bigg/ \\
& \left( c^2 (b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1+\text{Tanh}[\frac{x}{2}])^2 \right) \left( b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh}[\frac{x}{2}] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh}[\frac{x}{2}]^2 \right)} \right) \Bigg) - \\
& \left( 4bc^2 \left( (-b+c+\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\text{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\text{Tanh}[\frac{x}{2}])}} \right], 1 \right] - 2c \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \text{EllipticPi} \left[ \frac{(b+c-\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right. \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right]\right) \right] \Big/ \\
& \left( (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2)\right)} \right) - \\
& \left( 4b(b^2-c^2) \left(-b+c+\sqrt{b^2-c^2}\right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right. \\
& \left. \text{EllipticPi} \left[ \frac{(b+c-\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right) \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right]\right) \right] \Big/ \\
& \left( (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2)\right)} \right) + \\
& \left( 4b^3(b^2-c^2) \left(-b+c+\sqrt{b^2-c^2}\right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \text{EllipticPi} \left[ \frac{(b+c-\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right. \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right]\right)} \right) / \\
& \left( c^2 (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2)\right)} \right) + \\
& \left( 8b^3 \left( (-b+c-\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right. \right. \\
& \left. \left. \text{EllipticPi} \left[ \frac{(b+c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right) \right. \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right]\right)} \right) / \\
& \left( (-b-c-\sqrt{b^2-c^2}) (b-c+\sqrt{b^2-c^2}) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2)\right)} \right) - \\
& \left( 4b^5 \left( (-b+c-\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \text{EllipticPi} \left[ \frac{(b+c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right. \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right]\right)} \right) / \\
& \left( c^2 (-b-c-\sqrt{b^2-c^2}) (b-c+\sqrt{b^2-c^2}) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2)\right)} \right) - \\
& \left( 4bc^2 \left( (-b+c-\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right. \right. \\
& \left. \left. \text{EllipticPi} \left[ \frac{(b+c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right) \right. \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right]\right)} \right) / \\
& \left( (-b-c-\sqrt{b^2-c^2}) (b-c+\sqrt{b^2-c^2}) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2)\right)} \right) + \\
& \left( 8b^2\sqrt{b^2-c^2} \left( (-b+c-\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \text{EllipticPi} \left[ \frac{(b+c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right. \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right] \right) \right] \right) / \\
& \left( (-b-c-\sqrt{b^2-c^2}) (b-c+\sqrt{b^2-c^2}) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1 + \text{Tanh} \left[\frac{x}{2}\right])^2 \right) (b - \sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b + \sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2) \right)} \right) - \\
& \left( 8b^4 \sqrt{b^2-c^2} \left( (-b+c-\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right. \right. \\
& \left. \left. \text{EllipticPi} \left[ \frac{(b+c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right) \right) \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right] \right) \right] \right) / \\
& \left( c^2 (-b-c-\sqrt{b^2-c^2}) (b-c+\sqrt{b^2-c^2}) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1 + \text{Tanh} \left[\frac{x}{2}\right])^2 \right) (b - \sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b + \sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2) \right)} \right) + \\
& \left( 4b(b^2-c^2) \left( (-b+c-\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \text{EllipticPi} \left[ \frac{(b+c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right. \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right]\right)} \right] / \\
& \left( (-b-c-\sqrt{b^2-c^2}) (b-c+\sqrt{b^2-c^2}) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2)\right)} \right) - \\
& \left( 4b^3 (b^2-c^2) \left( (-b+c-\sqrt{b^2-c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] - 2c \right. \right. \\
& \left. \left. \text{EllipticPi} \left[ \frac{(b+c+\sqrt{b^2-c^2}) \left(1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 - \frac{c}{b+\sqrt{b^2-c^2}}\right)}, \text{ArcSin} \left[ \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \right], 1 \right] \right) \right. \\
& \left. (-1 + \text{Tanh} \left[\frac{x}{2}\right]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2}) \left(1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2}) \left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \text{Tanh} \left[\frac{x}{2}\right]\right)} \right] / \\
& \left( c^2 (-b-c-\sqrt{b^2-c^2}) (b-c+\sqrt{b^2-c^2}) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \text{Tanh} \left[\frac{x}{2}\right]\right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \text{Tanh} \left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \text{Tanh} \left[\frac{x}{2}\right]^2)\right)} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 2 b^3 \left( -1 + \frac{c}{b + \sqrt{b^2 - c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right], 1 \right] \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \left( \frac{c}{b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right) / \left( c \left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( b - \sqrt{(b - c)(b + c)} + 2 c \text{Tanh} \left[ \frac{x}{2} \right] + \left( b + \sqrt{(b - c)(b + c)} \right) \text{Tanh} \left[ \frac{x}{2} \right]^2 \right)} \right) \right) + \\
& \left( 2 b c \left( -1 + \frac{c}{b + \sqrt{b^2 - c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right], 1 \right] \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \left( \frac{c}{b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right) / \left( \left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( b - \sqrt{(b - c)(b + c)} + 2 c \text{Tanh} \left[ \frac{x}{2} \right] + \left( b + \sqrt{(b - c)(b + c)} \right) \text{Tanh} \left[ \frac{x}{2} \right]^2 \right)} \right) \right) - \\
& \left( 2 b^2 \sqrt{b^2 - c^2} \left( -1 + \frac{c}{b + \sqrt{b^2 - c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right], 1 \right] \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \left( \frac{c}{b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right) / \left( c \left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left( b c \left( 2 \left( \frac{1}{2} \left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] + \right. \right. \\
& \left. \left. \frac{c \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \operatorname{EllipticPi}\left[\frac{1-\frac{c}{b+\sqrt{b^2-c^2}}}{-1-\frac{c}{b+\sqrt{b^2-c^2}}}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right)} \\
& \left. \frac{\left(b + \sqrt{b^2 - c^2}\right) \left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(b + \sqrt{b^2 - c^2}\right) \left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right)}{\right)} \\
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) + \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \left( \frac{c}{b+\sqrt{b^2-c^2}} + \right. \\
& \left. \left. \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right) \left/ \left( \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right) \right) + \\
& \left( c \sqrt{b^2 - c^2} \left( 2 \left( \frac{1}{2} \left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] + \right. \right. \right.
\end{aligned}$$



$$\left. \frac{\begin{aligned} & c \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2 c \operatorname{EllipticPi}\left[\frac{1-\frac{c}{b+\sqrt{b^2-c^2}}}{-1-\frac{c}{b+\sqrt{b^2-c^2}}}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \\ & \frac{(b+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{\quad} \quad \frac{(b+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{\quad} \end{aligned}}{\begin{aligned} & (-1+\operatorname{Tanh}[\frac{x}{2}]) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}[\frac{x}{2}] \right) + \left(1+\operatorname{Tanh}[\frac{x}{2}]\right) \left( \frac{c}{b+\sqrt{b^2-c^2}} + \right. \\ & \left. \left. \operatorname{Tanh}[\frac{x}{2}] \right)^2 \right) \left/ \left( \sqrt{\left( (-1+\operatorname{Tanh}[\frac{x}{2}])^2 (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}[\frac{x}{2}] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}]^2 \right)} \right) \right/ \\ & \left. \left( (b-c)(b+c) (-1+\operatorname{Tanh}[\frac{x}{2}])^2 \sqrt{b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}[\frac{x}{2}] + b \operatorname{Tanh}[\frac{x}{2}]^2 + \sqrt{(b-c)(b+c)} \operatorname{Tanh}[\frac{x}{2}]^2} \right. \right. \\ & \left. \left. \sqrt{2c \operatorname{Tanh}[\frac{x}{2}] + \sqrt{b^2-c^2} (-1+\operatorname{Tanh}[\frac{x}{2}])^2 + b \left(1+\operatorname{Tanh}[\frac{x}{2}]\right)^2} \right) \right) \end{aligned}}$$

**Problem 775:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( -\sqrt{b^2-c^2} + b \cosh[x] + c \sinh[x] \right)^{3/2} dx$$

Optimal (type 3, 96 leaves, 2 steps):

$$-\frac{8\sqrt{b^2-c^2}(c\cosh[x]+b\sinh[x])}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh[x]+c\sinh[x]}}+\frac{2}{3}(c\cosh[x]+b\sinh[x])\sqrt{-\sqrt{b^2-c^2}+b\cosh[x]+c\sinh[x]}$$

Result (type 4, 9861 leaves):

$$-\frac{2b\sqrt{b^2-c^2}\sqrt{-\sqrt{b^2-c^2}+b\cosh[x]+c\sinh[x]}}{c}+$$

$$\left(-\frac{2b\sqrt{b^2-c^2}}{3c}+\frac{2}{3}c\cosh[x]+\frac{2}{3}b\sinh[x]\right)\sqrt{-\sqrt{b^2-c^2}+b\cosh[x]+c\sinh[x]}-32bc(-b+c)(b+c)(-b^2+c^2)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right]-2\operatorname{EllipticPi}\left[-1, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right]\right)$$

$$\sqrt{-\sqrt{(b-c)(b+c)}+b\cosh[x]+c\sinh[x]}\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)$$

$$\left(-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}\right)^{3/2}\left(c+(b+\sqrt{b^2-c^2})\operatorname{Tanh}\left[\frac{x}{2}\right]\right)\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\Bigg/$$

$$\left(3(b+c+\sqrt{b^2-c^2})^3(-b^2+c^2+b\sqrt{b^2-c^2})(1+\cosh[x])\sqrt{\frac{-\sqrt{(b-c)(b+c)}+b\cosh[x]+c\sinh[x]}{(1+\cosh[x])^2}}\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2\right.$$

$$\left.\sqrt{-\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\left(2c\operatorname{Tanh}\left[\frac{x}{2}\right]+\sqrt{b^2-c^2}\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)+b\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)}\right)+$$

$$\frac{1}{3c(1+\cosh[x])\sqrt{\frac{-\sqrt{(b-c)(b+c)}+b\cosh[x]+c\sinh[x]}{(1+\cosh[x])^2}}}\frac{8(b-c)(b+c)\sqrt{b^2-c^2}\sqrt{-\sqrt{(b-c)(b+c)}+b\cosh[x]+c\sinh[x]}}$$

$$\left( \left( b - c \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{b - \sqrt{b^2 - c^2} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{b^2 - c^2} \operatorname{Tanh}\left[\frac{x}{2}\right]^2} \right.$$

$$\left. \sqrt{-\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) + b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right) /$$

$$\left( (-b^2 + c^2) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) + b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) -$$

$$\left( \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{(b-c)(b+c)} \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right.$$

$$\left. \sqrt{-\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) + b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right)$$

$$\left( 2c^2 \left(-1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left( -\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] + 2 \right. \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{1 - \frac{c}{b + \sqrt{b^2 - c^2}}}{-1 - \frac{c}{b + \sqrt{b^2 - c^2}}}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}}\right], 1\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)$$

$$\left. \sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])}} \left(\frac{c}{b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \right) / \left( \left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \right)$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \left(8b^3 \left(-b + c + \sqrt{b^2 - c^2}\right)\right. \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)},\right. \\
& \left.\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}} \\
& \left.\left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)\right) \left/\left(\left(b-c-\sqrt{b^2-c^2}\right)\left(-b-c+\sqrt{b^2-c^2}\right)\left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right)\right)\right. \\
& \left.\left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right) \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \right. \\
& \left. \left(4b^5 \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] \right) \right. \\
& \left. \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)\right) \left/\right. \\
& \left.\left(c^2 \left(b-c-\sqrt{b^2-c^2}\right)\left(-b-c+\sqrt{b^2-c^2}\right)\left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right)\left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right)\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(4bc^2 \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(\left(b-c-\sqrt{b^2-c^2}\right) \left(-b-c+\sqrt{b^2-c^2}\right) \left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(4b(b^2 - c^2) \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(\left(b-c-\sqrt{b^2-c^2}\right) \left(-b-c+\sqrt{b^2-c^2}\right) \left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left(4b^3(b^2 - c^2) \left( \left(-b + c + \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(c^2(b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left(-\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left(8b^3 \left( \left(-b + c - \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(\left(-b-c-\sqrt{b^2-c^2}\right) \left(b-c+\sqrt{b^2-c^2}\right) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(4b^5 \left(-b + c - \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(c^2 \left(-b - c - \sqrt{b^2 - c^2}\right) \left(b - c + \sqrt{b^2 - c^2}\right) \left(\frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c}\right) \left(1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \right. \\
& \left(4b^2 \left(-b + c - \sqrt{b^2 - c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right], 1\right] \right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \\
& \left(\left(-b - c - \sqrt{b^2 - c^2}\right) \left(b - c + \sqrt{b^2 - c^2}\right) \left(\frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c}\right) \left(1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left(8b^2\sqrt{b^2-c^2} \left(-b+c-\sqrt{b^2-c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right]\right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \Bigg/ \\
& \left(\left(-b-c-\sqrt{b^2-c^2}\right) \left(b-c+\sqrt{b^2-c^2}\right) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(8b^4\sqrt{b^2-c^2} \left(-b+c-\sqrt{b^2-c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}\right], 1\right]\right) \\
& \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \Bigg/ \\
& \left(c^2 \left(-b-c-\sqrt{b^2-c^2}\right) \left(b-c+\sqrt{b^2-c^2}\right) \left(\frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c}\right) \left(1 + \frac{c}{b+\sqrt{b^2-c^2}}\right)\right)
\end{aligned}$$



$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left(4b(b^2 - c^2) \left( (-b + c - \sqrt{b^2 - c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \\
& \quad \left. (-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \right) / \\
& \left( (-b - c - \sqrt{b^2 - c^2}) (b - c + \sqrt{b^2 - c^2}) \left(\frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c}\right) \left(1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(4b^3(b^2 - c^2) \left( (-b + c - \sqrt{b^2 - c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \\
& \quad \left. (-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left(\frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \right) / \\
& \left(c^2 (-b - c - \sqrt{b^2 - c^2}) (b - c + \sqrt{b^2 - c^2}) \left(\frac{-b - \sqrt{b^2 - c^2}}{c} - \frac{-b + \sqrt{b^2 - c^2}}{c}\right) \left(1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(2b^3 \left(-1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{\left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}\right], 1\right] \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \right. \\
& \left. \sqrt{\frac{\left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{\left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(\frac{c}{b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \right) / \left(c \left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left(2bc \left(-1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{\left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}\right], 1\right] \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \right. \\
& \left. \sqrt{\frac{\left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{\left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(\frac{c}{b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \right) / \left(\left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b-c)(b+c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} - \\
& \left(2b^2 \sqrt{b^2 - c^2} \left(-1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{\left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}\right], 1\right] \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{\left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(\frac{c}{b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \Bigg/ \left(c \left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left(1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right)\right) \\
& \sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(b - \sqrt{(b - c)(b + c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b - c)(b + c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} + \\
& \left( b c \left( 2 \left( \frac{1}{2} \left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b + c + \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b - c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right] + \right. \right. \\
& \left. \left. \frac{c \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b + c + \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b - c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right] - 2c \operatorname{EllipticPi}\left[\frac{1 - \frac{c}{b + \sqrt{b^2 - c^2}}}{-1 - \frac{c}{b + \sqrt{b^2 - c^2}}}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b + c + \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b - c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}}, 1\right], 1\right]}{\left(b + \sqrt{b^2 - c^2}\right) \left(-1 - \frac{c}{b + \sqrt{b^2 - c^2}}\right)} \right) \right) \\
& \left. \left. \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{-\frac{(b + c + \sqrt{b^2 - c^2}) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b - c + \sqrt{b^2 - c^2}) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \left(\frac{c}{b + \sqrt{b^2 - c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) + \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \left(\frac{c}{b + \sqrt{b^2 - c^2}} + \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right) \right) \Bigg/ \left(\sqrt{\left(\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2 \left(b - \sqrt{(b - c)(b + c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left(b + \sqrt{(b - c)(b + c)}\right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)}\right) +
\end{aligned}$$



**Problem 776: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \, dx$$

Optimal (type 3, 39 leaves, 1 step):

$$\frac{2 (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{\sqrt{-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}$$

Result (type 4, 9771 leaves):

$$\frac{2 b \sqrt{-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}}{c} - \left( 8 b c \sqrt{b^2 - c^2} (-b^2 + c^2) \right. \\ \left. \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] - 2 \operatorname{EllipticPi}\left[-1, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}}\right]}, 1\right] \right) \\ \sqrt{-\sqrt{(b-c)(b+c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \\ \left( -\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \right)^{3/2} \left( c + (b+\sqrt{b^2-c^2}) \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right) / \\ \left( (b+c+\sqrt{b^2-c^2})^3 (-b^2+c^2+b\sqrt{b^2-c^2}) (1+\operatorname{Cosh}[x]) \sqrt{\frac{-\sqrt{(b-c)(b+c)} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}{(1+\operatorname{Cosh}[x])^2}} \right. \\ \left. \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)^2 \sqrt{-\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left( 2 c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) + b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right)} \right) -$$

$$\frac{1}{c(1 + \text{Cosh}[x])} \sqrt{\frac{-\sqrt{(b-c)(b+c)} + b \text{Cosh}[x] + c \text{Sinh}[x]}{(1 + \text{Cosh}[x])^2}} \cdot 2(b-c)(b+c) \sqrt{-\sqrt{(b-c)(b+c)} + b \text{Cosh}[x] + c \text{Sinh}[x]}$$

$$\left( \left( (b - c \text{Tanh}\left[\frac{x}{2}\right]) \sqrt{b - \sqrt{b^2 - c^2} + 2c \text{Tanh}\left[\frac{x}{2}\right] + b \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{b^2 - c^2} \text{Tanh}\left[\frac{x}{2}\right]^2} \right. \right. \\ \left. \left. \sqrt{-\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) + b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right) / \right. \\ \left. \left( (-b^2 + c^2) \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) + b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) - \right. \\ \left. \sqrt{\left(\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(b - \sqrt{(b-c)(b+c)} + 2c \text{Tanh}\left[\frac{x}{2}\right] + b \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{(b-c)(b+c)} \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right. \\ \left. \sqrt{-\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(2c \text{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) + b \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)\right)} \right) \\ \left( 2c^2 \left(-1 + \frac{c}{b + \sqrt{b^2 - c^2}}\right) \left( -\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \text{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \text{Tanh}\left[\frac{x}{2}\right])}}\right]}, 1\right] + 2 \right. \right.$$

$$\left. \text{EllipticPi}\left[\frac{1 - \frac{c}{b + \sqrt{b^2 - c^2}}}{-1 - \frac{c}{b + \sqrt{b^2 - c^2}}}, \text{ArcSin}\left[\sqrt{-\frac{(b+c + \sqrt{b^2 - c^2})(1 + \text{Tanh}\left[\frac{x}{2}\right])}{(b-c + \sqrt{b^2 - c^2})(-1 + \text{Tanh}\left[\frac{x}{2}\right])}}\right]}, 1\right] \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]\right) \right)$$

$$\begin{aligned}
& \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}\left(\frac{c}{b+\sqrt{b^2-c^2}}+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}\left/\left(\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)\left(1+\frac{c}{b+\sqrt{b^2-c^2}}\right)\right)\right. \\
& \sqrt{\left(\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2\left(b-\sqrt{(b-c)(b+c)}+2c\operatorname{Tanh}\left[\frac{x}{2}\right]+\left(b+\sqrt{(b-c)(b+c)}\right)\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)}+8b^3\left(-b+c+\sqrt{b^2-c^2}\right) \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}},1\right]-2c\operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)},\right. \\
& \left.\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}},1\right]\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}},\right. \\
& \left.\left(\frac{c}{b+\sqrt{b^2-c^2}}+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)\right]\left/\left(\left(b-c-\sqrt{b^2-c^2}\right)\left(-b-c+\sqrt{b^2-c^2}\right)\left(-\frac{-b-\sqrt{b^2-c^2}}{c}+\frac{-b+\sqrt{b^2-c^2}}{c}\right)\right)\right. \\
& \left.\left(1+\frac{c}{b+\sqrt{b^2-c^2}}\right)\sqrt{\left(\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)^2\left(b-\sqrt{(b-c)(b+c)}+2c\operatorname{Tanh}\left[\frac{x}{2}\right]+\left(b+\sqrt{(b-c)(b+c)}\right)\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\right)}- \right. \\
& \left.4b^5\left(-b+c+\sqrt{b^2-c^2}\right)\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}},1\right]-2c\right. \right. \\
& \left. \left.\operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)},\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}{(b-c+\sqrt{b^2-c^2})\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]\right)}},1\right]\right]\right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( c^2 (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} - \right. \\
& \left. \left( 4bc^2 \left( -b+c+\sqrt{b^2-c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right) \\
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( (b-c-\sqrt{b^2-c^2}) (-b-c+\sqrt{b^2-c^2}) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)^2 (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} - \right. \\
& \left. \left( 4b(b^2-c^2) \left( -b+c+\sqrt{b^2-c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( (b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 4b^3(b^2-c^2) \left( (-b+c+\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c-\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c-\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right) \\
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( c^2(b-c-\sqrt{b^2-c^2})(-b-c+\sqrt{b^2-c^2}) \left( -\frac{-b-\sqrt{b^2-c^2}}{c} + \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 8b^3 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1+\operatorname{Tanh}[\frac{x}{2}])^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}[\frac{x}{2}] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}])^2} \right) - \\
& \left( 4b^5 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right) \\
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( c^2 (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1+\operatorname{Tanh}[\frac{x}{2}])^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}[\frac{x}{2}] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}])^2} \right) - \\
& \left( 4bc^2 \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \\
& \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1+\operatorname{Tanh}[\frac{x}{2}])^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}[\frac{x}{2}] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}])^2} \right) + \\
& \left( 8b^2\sqrt{b^2-c^2} \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right) \\
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \\
& \left( (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1+\operatorname{Tanh}[\frac{x}{2}])^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}[\frac{x}{2}] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}[\frac{x}{2}])^2} \right) - \\
& \left( 8b^4\sqrt{b^2-c^2} \left( (-b+c-\sqrt{b^2-c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( c^2 \left( -b-c-\sqrt{b^2-c^2} \right) \left( b-c+\sqrt{b^2-c^2} \right) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)^2 \left( b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left( b + \sqrt{(b-c)(b+c)} \right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) + \\
& \left( 4b(b^2-c^2) \left( -b+c-\sqrt{b^2-c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right) \\
& \left. \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right) / \\
& \left( \left( -b-c-\sqrt{b^2-c^2} \right) \left( b-c+\sqrt{b^2-c^2} \right) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)^2 \left( b - \sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \left( b + \sqrt{(b-c)(b+c)} \right) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \right)} \right) - \\
& \left( 4b^3(b^2-c^2) \left( -b+c-\sqrt{b^2-c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] - 2c \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+c+\sqrt{b^2-c^2})\left(1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}{(b-c+\sqrt{b^2-c^2})\left(-1-\frac{c}{b+\sqrt{b^2-c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}[\frac{x}{2}])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}[\frac{x}{2}])}} \right], 1\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \sqrt{-\frac{(b+c+\sqrt{b^2-c^2})(1+\operatorname{Tanh}\left[\frac{x}{2}\right])}{(b-c+\sqrt{b^2-c^2})(-1+\operatorname{Tanh}\left[\frac{x}{2}\right])} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \\
& \left( c^2 (-b-c-\sqrt{b^2-c^2})(b-c+\sqrt{b^2-c^2}) \left( \frac{-b-\sqrt{b^2-c^2}}{c} - \frac{-b+\sqrt{b^2-c^2}}{c} \right) \left( 1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1+\operatorname{Tanh}\left[\frac{x}{2}\right])^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2) \right)} \right) - \\
& \left( 2b^3 \left( -1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \frac{\left( -1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right], 1 \right] \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \left( c \left( -1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( 1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1+\operatorname{Tanh}\left[\frac{x}{2}\right])^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2) \right)} \right) + \\
& \left( 2bc \left( -1 + \frac{c}{b+\sqrt{b^2-c^2}} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \frac{\left( -1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right], 1 \right] \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \right. \\
& \left. \sqrt{\frac{\left( -1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( 1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)}{\left( 1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( -1 + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \left( \frac{c}{b+\sqrt{b^2-c^2}} + \operatorname{Tanh}\left[\frac{x}{2}\right] \right)} \right) / \left( \left( -1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \left( 1 - \frac{c}{b+\sqrt{b^2-c^2}} \right) \right. \\
& \left. \sqrt{\left( (-1+\operatorname{Tanh}\left[\frac{x}{2}\right])^2 \right) (b-\sqrt{(b-c)(b+c)} + 2c \operatorname{Tanh}\left[\frac{x}{2}\right] + (b+\sqrt{(b-c)(b+c)}) \operatorname{Tanh}\left[\frac{x}{2}\right]^2) \right)} \right) -
\end{aligned}$$

$$\left( 2 b^2 \sqrt{b^2 - c^2} \left( -1 + \frac{c}{b + \sqrt{b^2 - c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right], 1 \right] \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \right.$$

$$\left. \sqrt{\frac{\left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{\left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \left( \frac{c}{b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right)} \right) / \left( c \left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \right)$$

$$\sqrt{\left( \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)^2 \left( b - \sqrt{(b - c)(b + c)} + 2 c \text{Tanh} \left[ \frac{x}{2} \right] + \left( b + \sqrt{(b - c)(b + c)} \right) \text{Tanh} \left[ \frac{x}{2} \right]^2 \right) \right)} +$$

$$\left( b c \left( 2 \frac{1}{2} \left( 1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b + c + \sqrt{b^2 - c^2}) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{(b - c + \sqrt{b^2 - c^2}) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] + \right.$$

$$\left. \frac{c \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b + c + \sqrt{b^2 - c^2}) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{(b - c + \sqrt{b^2 - c^2}) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right] - 2 c \text{EllipticPi} \left[ \frac{1 - \frac{c}{b + \sqrt{b^2 - c^2}}}{-1 - \frac{c}{b + \sqrt{b^2 - c^2}}}, \text{ArcSin} \left[ \sqrt{-\frac{(b + c + \sqrt{b^2 - c^2}) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{(b - c + \sqrt{b^2 - c^2}) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}} \right], 1 \right]}{\left( b + \sqrt{b^2 - c^2} \right) \left( -1 - \frac{c}{b + \sqrt{b^2 - c^2}} \right)} \right)$$

$$\left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \sqrt{-\frac{(b + c + \sqrt{b^2 - c^2}) \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}{(b - c + \sqrt{b^2 - c^2}) \left( -1 + \text{Tanh} \left[ \frac{x}{2} \right] \right)}} \left( \frac{c}{b + \sqrt{b^2 - c^2}} + \text{Tanh} \left[ \frac{x}{2} \right] \right) + \left( 1 + \text{Tanh} \left[ \frac{x}{2} \right] \right) \left( \frac{c}{b + \sqrt{b^2 - c^2}} + \right)$$



$$\sqrt{2c \operatorname{Tanh}\left[\frac{x}{2}\right] + \sqrt{b^2 - c^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) + b \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)}$$

**Problem 777:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]}} dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{(b^2 - c^2)^{1/4} \operatorname{Sinh}[x + i \operatorname{ArcTan}[b, -i c]]}{\sqrt{2} \sqrt{-\sqrt{b^2 - c^2} + \sqrt{b^2 - c^2} \operatorname{Cosh}[x + i \operatorname{ArcTan}[b, -i c]]}}\right]}{(b^2 - c^2)^{1/4}}$$

Result (type 4, 52 609 leaves): Display of huge result suppressed!

**Problem 778:** Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^{3/2}} dx$$

Optimal (type 3, 159 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{(b^2 - c^2)^{1/4} \operatorname{Sinh}[x + i \operatorname{ArcTan}[b, -i c]]}{\sqrt{2} \sqrt{-\sqrt{b^2 - c^2} + \sqrt{b^2 - c^2} \operatorname{Cosh}[x + i \operatorname{ArcTan}[b, -i c]]}}\right]}{2 \sqrt{2} (b^2 - c^2)^{3/4}} - \frac{c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{2 \sqrt{b^2 - c^2} \left(-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???



**Problem 779: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\left(-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^{5/2}} dx$$

Optimal (type 3, 211 leaves, 5 steps):

$$\frac{3 \operatorname{ArcTanh}\left[\frac{(b^2 - c^2)^{1/4} \operatorname{Sinh}[x + i \operatorname{ArcTan}[b, -i c]]}{\sqrt{2} \sqrt{-\sqrt{b^2 - c^2} + \sqrt{b^2 - c^2}} \operatorname{Cosh}[x + i \operatorname{ArcTan}[b, -i c]]}\right]}{16 \sqrt{2} (b^2 - c^2)^{5/4}} - \frac{c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]}{4 \sqrt{b^2 - c^2} \left(-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^{5/2}} + \frac{3 (c \operatorname{Cosh}[x] + b \operatorname{Sinh}[x])}{16 (b^2 - c^2) \left(-\sqrt{b^2 - c^2} + b \operatorname{Cosh}[x] + c \operatorname{Sinh}[x]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

**Problem 846: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 \operatorname{Csch}[x] \operatorname{Sech}[x]}{\sqrt{a \operatorname{Sech}[x]^4}} dx$$

Optimal (type 4, 98 leaves, 6 steps):

$$-\frac{x^3 \operatorname{Sech}[x]^2}{3 \sqrt{a \operatorname{Sech}[x]^4}} + \frac{x^2 \operatorname{Log}[1 - e^{2x}] \operatorname{Sech}[x]^2}{\sqrt{a \operatorname{Sech}[x]^4}} + \frac{x \operatorname{PolyLog}[2, e^{2x}] \operatorname{Sech}[x]^2}{\sqrt{a \operatorname{Sech}[x]^4}} - \frac{\operatorname{PolyLog}[3, e^{2x}] \operatorname{Sech}[x]^2}{2 \sqrt{a \operatorname{Sech}[x]^4}}$$

Result (type 4, 65 leaves):

$$\frac{(i \pi^3 - 8 x^3 + 24 x^2 \operatorname{Log}[1 - e^{2x}] + 24 x \operatorname{PolyLog}[2, e^{2x}] - 12 \operatorname{PolyLog}[3, e^{2x}]) \operatorname{Sech}[x]^2}{24 \sqrt{a \operatorname{Sech}[x]^4}}$$

**Problem 852: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 \operatorname{Csch}[x] \operatorname{Sech}[x] \sqrt{a \operatorname{Sech}[x]^4} dx$$

Optimal (type 4, 204 leaves, 16 steps):

$$\begin{aligned} & \frac{1}{2} x^2 \operatorname{Cosh}[x]^2 \sqrt{a \operatorname{Sech}[x]^4} - 2 x^2 \operatorname{ArcTanh}[e^{2x}] \operatorname{Cosh}[x]^2 \sqrt{a \operatorname{Sech}[x]^4} + \operatorname{Cosh}[x]^2 \operatorname{Log}[\operatorname{Cosh}[x]] \sqrt{a \operatorname{Sech}[x]^4} - \\ & x \operatorname{Cosh}[x]^2 \operatorname{PolyLog}[2, -e^{2x}] \sqrt{a \operatorname{Sech}[x]^4} + x \operatorname{Cosh}[x]^2 \operatorname{PolyLog}[2, e^{2x}] \sqrt{a \operatorname{Sech}[x]^4} + \frac{1}{2} \operatorname{Cosh}[x]^2 \operatorname{PolyLog}[3, -e^{2x}] \sqrt{a \operatorname{Sech}[x]^4} - \\ & \frac{1}{2} \operatorname{Cosh}[x]^2 \operatorname{PolyLog}[3, e^{2x}] \sqrt{a \operatorname{Sech}[x]^4} - x \operatorname{Cosh}[x] \sqrt{a \operatorname{Sech}[x]^4} \operatorname{Sinh}[x] - \frac{1}{2} x^2 \sqrt{a \operatorname{Sech}[x]^4} \operatorname{Sinh}[x]^2 \end{aligned}$$

Result (type 4, 120 leaves):

$$\begin{aligned} & \frac{1}{24} \operatorname{Cosh}[x]^2 \sqrt{a \operatorname{Sech}[x]^4} \left( i \pi^3 - 16 x^3 - 24 x^2 \operatorname{Log}[1 + e^{-2x}] + 24 x^2 \operatorname{Log}[1 - e^{2x}] + 24 \operatorname{Log}[\operatorname{Cosh}[x]] + \right. \\ & \left. 24 x \operatorname{PolyLog}[2, -e^{-2x}] + 24 x \operatorname{PolyLog}[2, e^{2x}] + 12 \operatorname{PolyLog}[3, -e^{-2x}] - 12 \operatorname{PolyLog}[3, e^{2x}] + 12 x^2 \operatorname{Sech}[x]^2 - 24 x \operatorname{Tanh}[x] \right) \end{aligned}$$

**Problem 869: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{x}{a + b \operatorname{Cosh}[x] \operatorname{Sinh}[x]} dx$$

Optimal (type 4, 186 leaves, 9 steps):

$$\frac{x \operatorname{Log}\left[1 + \frac{b e^{2x}}{2a - \sqrt{4a^2 + b^2}}\right]}{\sqrt{4a^2 + b^2}} - \frac{x \operatorname{Log}\left[1 + \frac{b e^{2x}}{2a + \sqrt{4a^2 + b^2}}\right]}{\sqrt{4a^2 + b^2}} + \frac{\operatorname{PolyLog}\left[2, -\frac{b e^{2x}}{2a - \sqrt{4a^2 + b^2}}\right]}{2\sqrt{4a^2 + b^2}} - \frac{\operatorname{PolyLog}\left[2, -\frac{b e^{2x}}{2a + \sqrt{4a^2 + b^2}}\right]}{2\sqrt{4a^2 + b^2}}$$

Result (type 4, 960 leaves):

$$\begin{aligned}
& \frac{1}{2} \left( -\frac{i \pi \operatorname{ArcTanh}\left[\frac{-b+2a \operatorname{Tanh}[x]}{\sqrt{4a^2+b^2}}\right]}{\sqrt{4a^2+b^2}} - \right. \\
& \frac{1}{\sqrt{-4a^2-b^2}} \left( 2 \operatorname{ArcCos}\left[-\frac{2ia}{b}\right] \operatorname{ArcTanh}\left[\frac{(2a+ib) \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right]}{\sqrt{-4a^2-b^2}}\right] + (\pi-4ix) \operatorname{ArcTanh}\left[\frac{(2a-ib) \operatorname{Tan}\left[\frac{1}{4}(\pi+4ix)\right]}{\sqrt{-4a^2-b^2}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{2ia}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(2a+ib) \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right]}{\sqrt{-4a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{(2ia+b) (-2ia+b+\sqrt{-4a^2-b^2}) (1+i \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right])}{b (2ia+b+i\sqrt{-4a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right])}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{2ia}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(2a+ib) \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right]}{\sqrt{-4a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{(2ia+b) (2ia-b+\sqrt{-4a^2-b^2}) (i+\operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right])}{b (2a-ib+\sqrt{-4a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right])}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{2ia}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(2a+ib) \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right]}{\sqrt{-4a^2-b^2}}\right] - 2i \operatorname{ArcTanh}\left[\frac{(2a-ib) \operatorname{Tan}\left[\frac{1}{4}(\pi+4ix)\right]}{\sqrt{-4a^2-b^2}}\right] \right) \right. \\
& \left. \operatorname{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right) \sqrt{-4a^2-b^2} e^{-x}}{\sqrt{2} \sqrt{-ib} \sqrt{a+b \operatorname{Cosh}[x]} \operatorname{Sinh}[x]}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{2ia}{b}\right] + 2i \left( \operatorname{ArcTanh}\left[\frac{(2a+ib) \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right]}{\sqrt{-4a^2-b^2}}\right] + \operatorname{ArcTanh}\left[\frac{(2a-ib) \operatorname{Tan}\left[\frac{1}{4}(\pi+4ix)\right]}{\sqrt{-4a^2-b^2}}\right] \right) \right) \right. \\
& \left. \operatorname{Log}\left[\frac{(-1)^{1/4} \sqrt{-4a^2-b^2} e^x}{2 \sqrt{-ib} \sqrt{a+b \operatorname{Cosh}[x]} \operatorname{Sinh}[x]}\right] + i \left( \operatorname{PolyLog}\left[2, \frac{(2ia+\sqrt{-4a^2-b^2}) (2ia+b-i\sqrt{-4a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right])}{b (2ia+b+i\sqrt{-4a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right])}\right] - \right. \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{(2a+i\sqrt{-4a^2-b^2}) (-2a+ib+\sqrt{-4a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right])}{b (2ia+b+i\sqrt{-4a^2-b^2} \operatorname{Cot}\left[\frac{1}{4}(\pi+4ix)\right])}\right] \right) \right) \right)
\end{aligned}$$

**Problem 871: Unable to integrate problem.**

$$\int F^{c(a+bx)} \operatorname{Sinh}[d+ex]^n dx$$

Optimal (type 5, 95 leaves, 2 steps):

$$-\frac{1}{e^{n-bc \operatorname{Log}[F]}} (1-e^{2(d+ex)})^{-n} F^{c(a+bx)} \operatorname{Hypergeometric2F1}\left[-n, -\frac{e^{n-bc \operatorname{Log}[F]}}{2e}, \frac{1}{2} \left(2-n + \frac{bc \operatorname{Log}[F]}{e}\right), e^{2(d+ex)}\right] \operatorname{Sinh}[d+ex]^n$$

Result (type 8, 20 leaves):

$$\int F^{c(a+bx)} \operatorname{Sinh}[d+ex]^n dx$$

**Problem 882: Result more than twice size of optimal antiderivative.**

$$\int e^{c+dx} \operatorname{Csch}[a+bx]^2 dx$$

Optimal (type 5, 54 leaves, 1 step):

$$\frac{4 e^{c+dx+2(a+bx)} \operatorname{Hypergeometric2F1}\left[2, 1 + \frac{d}{2b}, 2 + \frac{d}{2b}, e^{2(a+bx)}\right]}{2b+d}$$

Result (type 5, 133 leaves):

$$\frac{1}{b} e^c \left( -\frac{1}{(2b+d)(-1+e^{2a})} \right. \\ \left. 2 e^{2a} \left( (2b+d) e^{dx} \operatorname{Hypergeometric2F1}\left[1, \frac{d}{2b}, 1 + \frac{d}{2b}, e^{2(a+bx)}\right] - d e^{(2b+d)x} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{d}{2b}, 2 + \frac{d}{2b}, e^{2(a+bx)}\right] \right) + e^{dx} \right. \\ \left. \operatorname{Csch}[a] \operatorname{Csch}[a+bx] \operatorname{Sinh}[bx] \right)$$

**Problem 884: Unable to integrate problem.**

$$\int F^{c(a+bx)} \operatorname{Cosh}[d+ex]^n dx$$

Optimal (type 5, 95 leaves, 2 steps):

$$-\frac{1}{e^{n-bc \operatorname{Log}[F]}} (1+e^{2(d+ex)})^{-n} F^{c(a+bx)} \operatorname{Cosh}[d+ex]^n \operatorname{Hypergeometric2F1}\left[-n, -\frac{en-bc \operatorname{Log}[F]}{2e}, \frac{1}{2} \left(2-n + \frac{bc \operatorname{Log}[F]}{e}\right), -e^{2(d+ex)}\right]$$

Result (type 8, 20 leaves):

$$\int F^{c(a+bx)} \operatorname{Cosh}[d+ex]^n dx$$

**Problem 889: Result more than twice size of optimal antiderivative.**

$$\int e^{a+bx} \operatorname{Sech}[c+dx]^2 dx$$

Optimal (type 5, 56 leaves, 1 step):

$$\frac{4 e^{a+bx+2(c+dx)} \operatorname{Hypergeometric2F1}\left[2, 1 + \frac{b}{2d}, 2 + \frac{b}{2d}, -e^{2(c+dx)}\right]}{b + 2d}$$

Result (type 5, 138 leaves):

$$-\frac{2 b e^{a+2c} \left( \frac{e^{(b+2d)x} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{b}{2d}, 2 + \frac{b}{2d}, -e^{2(c+dx)}\right]}{b+2d} - \frac{e^{bx} \operatorname{Hypergeometric2F1}\left[1, \frac{b}{2d}, 1 + \frac{b}{2d}, -e^{2(c+dx)}\right]}{b} \right)}{d(1 + e^{2c})} + \frac{e^{a+bx} \operatorname{Sech}[c] \operatorname{Sech}[c+dx] \operatorname{Sinh}[dx]}{d}$$

**Problem 891: Unable to integrate problem.**

$$\int F^{c(a+bx)} \operatorname{Sech}[d+ex]^n dx$$

Optimal (type 5, 90 leaves, 2 steps):

$$\frac{(1 + e^{2(d+ex)})^n F^{a+bcx} \operatorname{Hypergeometric2F1}\left[n, \frac{en+bc \operatorname{Log}[F]}{2e}, 1 + \frac{en+bc \operatorname{Log}[F]}{2e}, -e^{2(d+ex)}\right] \operatorname{Sech}[d+ex]^n}{en + bc \operatorname{Log}[F]}$$

Result (type 8, 20 leaves):

$$\int F^{c(a+bx)} \operatorname{Sech}[d+ex]^n dx$$

**Problem 892: Unable to integrate problem.**

$$\int F^{c(a+bx)} \operatorname{Csch}[d+ex]^n dx$$

Optimal (type 5, 91 leaves, 2 steps):

$$-\frac{(1 - e^{-2(d+ex)})^n F^{a+bcx} \operatorname{Csch}[d+ex]^n \operatorname{Hypergeometric2F1}\left[n, \frac{en-bc \operatorname{Log}[F]}{2e}, \frac{1}{2} \left(2 + n - \frac{bc \operatorname{Log}[F]}{e}\right), e^{-2(d+ex)}\right]}{en - bc \operatorname{Log}[F]}$$

Result (type 8, 20 leaves):

$$\int F^{c(a+bx)} \operatorname{Csch}[d+ex]^n dx$$

**Problem 899: Result more than twice size of optimal antiderivative.**

$$\int \frac{F^{c(a+bx)}}{f + f \operatorname{Cosh}[d+ex]} dx$$

Optimal (type 5, 61 leaves, 2 steps):

$$\frac{2 e^{d+e x} F^{c(a+b x)} \text{Hypergeometric2F1}\left[2, 1 + \frac{b c \text{Log}[F]}{e}, 2 + \frac{b c \text{Log}[F]}{e}, -e^{d+e x}\right]}{f(e + b c \text{Log}[F])}$$

Result (type 5, 213 leaves):

$$\frac{1}{e f (1 + \text{Cosh}[d + e x]) (e + b c \text{Log}[F])} 2 F^{-\frac{b c d}{e}} \text{Cosh}\left[\frac{1}{2}(d + e x)\right] \\ \left(-b c e^{\frac{(d+e x)(e+b c \text{Log}[F])}{e}} F^{a c} \text{Cosh}\left[\frac{1}{2}(d + e x)\right] \text{Hypergeometric2F1}\left[1, 1 + \frac{b c \text{Log}[F]}{e}, 2 + \frac{b c \text{Log}[F]}{e}, -e^{d+e x}\right] \text{Log}[F] + F^{c(a+b(\frac{d}{e}+x))} \text{Cosh}\left[\frac{1}{2}(d + e x)\right] \right. \\ \left. \text{Hypergeometric2F1}\left[1, \frac{b c \text{Log}[F]}{e}, 1 + \frac{b c \text{Log}[F]}{e}, -e^{d+e x}\right] (e + b c \text{Log}[F]) + F^{c(a+b(\frac{d}{e}+x))} (e + b c \text{Log}[F]) \text{Sinh}\left[\frac{1}{2}(d + e x)\right] \right)$$

Problem 900: Result more than twice size of optimal antiderivative.

$$\int \frac{F^{c(a+b x)}}{(f + f \text{Cosh}[d + e x])^2} dx$$

Optimal (type 5, 151 leaves, 3 steps):

$$\frac{2 e^{d+e x} F^{c(a+b x)} \text{Hypergeometric2F1}\left[2, 1 + \frac{b c \text{Log}[F]}{e}, 2 + \frac{b c \text{Log}[F]}{e}, -e^{d+e x}\right] (e - b c \text{Log}[F])}{3 e^2 f^2} + \\ \frac{b c F^{c(a+b x)} \text{Log}[F] \text{Sech}\left[\frac{d}{2} + \frac{e x}{2}\right]^2}{6 e^2 f^2} + \frac{F^{c(a+b x)} \text{Sech}\left[\frac{d}{2} + \frac{e x}{2}\right]^2 \text{Tanh}\left[\frac{d}{2} + \frac{e x}{2}\right]}{6 e f^2}$$

Result (type 5, 712 leaves):

$$\begin{aligned}
& \frac{2 b c F^{\frac{c(-bd+ae)}{e} + \frac{2bc\left(\frac{d+ex}{2}\right)}{e}} \operatorname{Cosh}\left[\frac{d}{2} + \frac{ex}{2}\right]^2 \operatorname{Log}[F]}{3 e^2 (f + f \operatorname{Cosh}[d + ex])^2} + \frac{1}{3 e^4 (f + f \operatorname{Cosh}[d + ex])^2} 8 b c F^{\frac{c(-bd+ae)}{e}} \operatorname{Cosh}\left[\frac{d}{2} + \frac{ex}{2}\right]^4 \operatorname{Log}[F] (-e + b c \operatorname{Log}[F]) (e + b c \operatorname{Log}[F]) \\
& \left( - \frac{e F^{a c - \frac{bcd}{e} - \frac{c(-bd+ae)}{e} + \frac{2bc\left(\frac{d+ex}{2}\right)}{e}}{2 b c \operatorname{Log}[F]} \operatorname{Hypergeometric2F1}\left[1, \frac{b c \operatorname{Log}[F]}{e}, 1 + \frac{b c \operatorname{Log}[F]}{e}, -e^2 \left(\frac{d+ex}{2}\right)\right] + \frac{1}{2 (e + b c \operatorname{Log}[F])} e^e \left( 2 + \frac{\left(\frac{d+ex}{2}\right) \left( a c - \frac{bcd}{e} - \frac{c(-bd+ae)}{e} + \frac{2bc\left(\frac{d+ex}{2}\right)}{e} \right) \operatorname{Log}[F]}{\frac{d+ex}{2}} \right) \right) \\
& \left( e^{2\left(\frac{d+ex}{2}\right)} \left( 1 + \frac{\left( a c - \frac{bcd}{e} - \frac{c(-bd+ae)}{e} + \frac{2bc\left(\frac{d+ex}{2}\right)}{e} \right) \operatorname{Log}[F]}{2\left(\frac{d+ex}{2}\right)} + \frac{1}{2} \left( -2 - \frac{\left( a c - \frac{bcd}{e} - \frac{c(-bd+ae)}{e} + \frac{2bc\left(\frac{d+ex}{2}\right)}{e} \right) \operatorname{Log}[F]}{\frac{d+ex}{2}} \right) \right) \operatorname{Hypergeometric2F1}\left[1, \frac{e + b c \operatorname{Log}[F]}{e}, 1 + \frac{e + b c \operatorname{Log}[F]}{e}, -e^2 \left(\frac{d+ex}{2}\right)\right] + \right) \\
& \frac{2 F^{\frac{c(-bd+ae)}{e} + \frac{2bc\left(\frac{d+ex}{2}\right)}{e}} \operatorname{Cosh}\left[\frac{d}{2} + \frac{ex}{2}\right] \operatorname{Sinh}\left[\frac{d}{2} + \frac{ex}{2}\right]}{3 e (f + f \operatorname{Cosh}[d + ex])^2} + \frac{4 F^{\frac{c(-bd+ae)}{e} + \frac{2bc\left(\frac{d+ex}{2}\right)}{e}} \operatorname{Cosh}\left[\frac{d}{2} + \frac{ex}{2}\right]^3 (e^2 - b^2 c^2 \operatorname{Log}[F]^2) \operatorname{Sinh}\left[\frac{d}{2} + \frac{ex}{2}\right]}{3 e^3 (f + f \operatorname{Cosh}[d + ex])^2}
\end{aligned}$$

Problem 937: Result is not expressed in closed-form.

$$\int e^x \operatorname{Sech}[2x] \operatorname{Tanh}[2x] dx$$

Optimal (type 3, 113 leaves, 12 steps):

$$-\frac{e^{3x}}{1+e^{4x}} - \frac{\operatorname{ArcTan}[1-\sqrt{2}e^x]}{2\sqrt{2}} + \frac{\operatorname{ArcTan}[1+\sqrt{2}e^x]}{2\sqrt{2}} + \frac{\operatorname{Log}[1-\sqrt{2}e^x+e^{2x}]}{4\sqrt{2}} - \frac{\operatorname{Log}[1+\sqrt{2}e^x+e^{2x}]}{4\sqrt{2}}$$

Result (type 7, 48 leaves):

$$-\frac{e^{3x}}{1+e^{4x}} - \frac{1}{4} \operatorname{RootSum}\left[1+\#1^4 \&, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1} \&\right]$$

### Problem 938: Result is not expressed in closed-form.

$$\int e^x \operatorname{Sech}[2x]^2 \operatorname{Tanh}[2x] dx$$

Optimal (type 3, 129 leaves, 13 steps):

$$-\frac{e^{5x}}{(1+e^{4x})^2} - \frac{e^x}{4(1+e^{4x})} - \frac{\operatorname{ArcTan}[1-\sqrt{2}e^x]}{8\sqrt{2}} + \frac{\operatorname{ArcTan}[1+\sqrt{2}e^x]}{8\sqrt{2}} - \frac{\operatorname{Log}[1-\sqrt{2}e^x+e^{2x}]}{16\sqrt{2}} + \frac{\operatorname{Log}[1+\sqrt{2}e^x+e^{2x}]}{16\sqrt{2}}$$

Result (type 7, 57 leaves):

$$-\frac{e^x(1+5e^{4x})}{4(1+e^{4x})^2} - \frac{1}{16} \operatorname{RootSum}\left[1+\#1^4 \&, \frac{x-\operatorname{Log}[e^x-\#1]}{\#1^3} \&\right]$$

### Problem 939: Result is not expressed in closed-form.

$$\int e^x \operatorname{Sech}[2x] \operatorname{Tanh}[2x]^2 dx$$

Optimal (type 3, 130 leaves, 13 steps):

$$\frac{e^{3x}}{(1+e^{4x})^2} - \frac{3e^{3x}}{4(1+e^{4x})} - \frac{5\operatorname{ArcTan}[1-\sqrt{2}e^x]}{8\sqrt{2}} + \frac{5\operatorname{ArcTan}[1+\sqrt{2}e^x]}{8\sqrt{2}} + \frac{5\operatorname{Log}[1-\sqrt{2}e^x+e^{2x}]}{16\sqrt{2}} - \frac{5\operatorname{Log}[1+\sqrt{2}e^x+e^{2x}]}{16\sqrt{2}}$$

Result (type 7, 58 leaves):

$$\frac{e^{3x}-3e^{7x}}{4(1+e^{4x})^2} - \frac{5}{16} \operatorname{RootSum}\left[1+\#1^4 \&, \frac{x-\operatorname{Log}[e^x-\#1]}{\#1} \&\right]$$

### Problem 940: Result is not expressed in closed-form.

$$\int e^x \operatorname{Sech}[2x]^2 \operatorname{Tanh}[2x]^2 dx$$

Optimal (type 3, 149 leaves, 14 steps):

$$\frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} - \frac{3e^x}{8(1+e^{4x})} - \frac{3\operatorname{ArcTan}[1-\sqrt{2}e^x]}{16\sqrt{2}} + \frac{3\operatorname{ArcTan}[1+\sqrt{2}e^x]}{16\sqrt{2}} - \frac{3\operatorname{Log}[1-\sqrt{2}e^x+e^{2x}]}{32\sqrt{2}} + \frac{3\operatorname{Log}[1+\sqrt{2}e^x+e^{2x}]}{32\sqrt{2}}$$

Result (type 7, 64 leaves):



$$\frac{1}{96} \left( -\frac{4 e^x (9 + 6 e^{4x} + 29 e^{8x})}{(1 + e^{4x})^3} - 9 \operatorname{RootSum} \left[ 1 + \#1^4 \&, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^3} \& \right] \right)$$

**Problem 949: Result more than twice size of optimal antiderivative.**

$$\int e^{c+dx} \operatorname{Coth}[a + bx] dx$$

Optimal (type 5, 53 leaves, 4 steps):

$$\frac{e^{c+dx}}{d} - \frac{2 e^{c+dx} \operatorname{Hypergeometric2F1} \left[ 1, \frac{d}{2b}, 1 + \frac{d}{2b}, e^{2(a+bx)} \right]}{d}$$

Result (type 5, 120 leaves):

$$\frac{e^{c+dx} \operatorname{Coth}[a]}{d} - \frac{2 e^{2a+c} \left( \frac{e^{dx} \operatorname{Hypergeometric2F1} \left[ 1, \frac{d}{2b}, 1 + \frac{d}{2b}, e^{2(a+bx)} \right]}{d} - \frac{e^{(2b+d)x} \operatorname{Hypergeometric2F1} \left[ 1, 1 + \frac{d}{2b}, 2 + \frac{d}{2b}, e^{2(a+bx)} \right]}{2b+d} \right)}{-1 + e^{2a}}$$

**Problem 985: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sech}[x]^2}{2 + 2 \operatorname{Tanh}[x] + \operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 5 leaves, 3 steps):

$$\operatorname{ArcTan}[1 + \operatorname{Tanh}[x]]$$

Result (type 3, 23 leaves):

$$\frac{1}{2} \left( -\operatorname{ArcTan}[\operatorname{Cosh}[x] (\operatorname{Cosh}[x] - \operatorname{Sinh}[x])] + \operatorname{ArcTan}[1 + \operatorname{Tanh}[x]] \right)$$

**Problem 994: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sech}[x]^2 \operatorname{Tanh}[x]^6 (1 - \operatorname{Tanh}[x]^2)^3 dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$\frac{\operatorname{Tanh}[x]^7}{7} - \frac{\operatorname{Tanh}[x]^9}{3} + \frac{3 \operatorname{Tanh}[x]^{11}}{11} - \frac{\operatorname{Tanh}[x]^{13}}{13}$$

Result (type 3, 67 leaves):

$$\frac{16 \operatorname{Tanh}[x]}{3003} + \frac{8 \operatorname{Sech}[x]^2 \operatorname{Tanh}[x]}{3003} + \frac{2 \operatorname{Sech}[x]^4 \operatorname{Tanh}[x]}{1001} +$$

$$\frac{5 \operatorname{Sech}[x]^6 \operatorname{Tanh}[x]}{3003} - \frac{53}{429} \operatorname{Sech}[x]^8 \operatorname{Tanh}[x] + \frac{27}{143} \operatorname{Sech}[x]^{10} \operatorname{Tanh}[x] - \frac{1}{13} \operatorname{Sech}[x]^{12} \operatorname{Tanh}[x]$$

**Problem 998: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sech}[x]^2}{\sqrt{4 - \operatorname{Sech}[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\operatorname{ArcSinh}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{3}}\right]$$

Result (type 3, 43 leaves):

$$\frac{\operatorname{ArcTanh}\left[\frac{\operatorname{Sinh}[x]}{\sqrt{1+2 \operatorname{Cosh}[2x]}}\right] \sqrt{1+2 \operatorname{Cosh}[2x]} \operatorname{Sech}[x]}{\sqrt{4 - \operatorname{Sech}[x]^2}}$$

**Problem 999: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sech}[x]^2}{\sqrt{1 - 4 \operatorname{Tanh}[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcSin}[2 \operatorname{Tanh}[x]]$$

Result (type 3, 52 leaves):

$$\frac{\operatorname{ArcTanh}\left[\frac{2\sqrt{2} \operatorname{Sinh}[x]}{\sqrt{-5+3 \operatorname{Cosh}[2x]}}\right] \sqrt{-5+3 \operatorname{Cosh}[2x]} \operatorname{Sech}[x]}{2 \sqrt{2 - 8 \operatorname{Tanh}[x]^2}}$$

**Problem 1000: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sech}[x]^2}{\sqrt{-4 + \operatorname{Tanh}[x]^2}} dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\operatorname{ArcTanh}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-4 + \operatorname{Tanh}[x]^2}}\right]$$

Result (type 3, 51 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{2} \operatorname{Sinh}[x]}{\sqrt{5+3 \operatorname{Cosh}[2x]}}\right] \sqrt{5+3 \operatorname{Cosh}[2x]} \operatorname{Sech}[x]}{\sqrt{2} \sqrt{-4 + \operatorname{Tanh}[x]^2}}$$

**Problem 1001: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{1 + \operatorname{Coth}[x]^2} \operatorname{Sech}[x]^2 dx$$

Optimal (type 3, 19 leaves, 3 steps):

$$-\operatorname{ArcSinh}[\operatorname{Coth}[x]] + \sqrt{1 + \operatorname{Coth}[x]^2} \operatorname{Tanh}[x]$$

Result (type 3, 51 leaves):

$$\sqrt{1 + \operatorname{Coth}[x]^2} \operatorname{Sech}[2x] \operatorname{Sinh}[x] \left( \operatorname{Cosh}[x] - \operatorname{ArcTan}\left[\frac{\operatorname{Cosh}[x]}{\sqrt{-\operatorname{Cosh}[2x]}}\right] \sqrt{-\operatorname{Cosh}[2x]} + \operatorname{Sinh}[x] \operatorname{Tanh}[x] \right)$$

**Problem 1002: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sech}[x]^2 \sqrt{1 + \operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{1}{2} \operatorname{ArcSinh}[\operatorname{Tanh}[x]] + \frac{1}{2} \operatorname{Tanh}[x] \sqrt{1 + \operatorname{Tanh}[x]^2}$$

Result (type 3, 49 leaves):

$$\frac{\left( \operatorname{ArcTanh} \left[ \frac{\operatorname{Sinh}[x]}{\sqrt{\operatorname{Cosh}[2x]}} \right] \operatorname{Cosh}[x] + \sqrt{\operatorname{Cosh}[2x]} \operatorname{Tanh}[x] \right) \sqrt{1 + \operatorname{Tanh}[x]^2}}{2 \sqrt{\operatorname{Cosh}[2x]}}$$

**Problem 1026: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cosh}[x]^3 (a + b \operatorname{Cosh}[x]^2)^3 \operatorname{Sinh}[x] dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{a (a + b \operatorname{Cosh}[x]^2)^4}{8 b^2} + \frac{(a + b \operatorname{Cosh}[x]^2)^5}{10 b^2}$$

Result (type 3, 136 leaves):

$$\frac{1}{32} \left( 12 a^2 b \operatorname{Cosh}[x]^4 + 8 a b^2 \operatorname{Cosh}[x]^6 + 2 b^3 \operatorname{Cosh}[x]^8 + 4 a^3 \operatorname{Cosh}[2x] + \right. \\ \left. 4 a^2 b \operatorname{Cosh}[x]^3 \operatorname{Cosh}[3x] + a^3 \operatorname{Cosh}[4x] + \frac{1}{32} a b^2 (48 \operatorname{Cosh}[2x] + 36 \operatorname{Cosh}[4x] + 16 \operatorname{Cosh}[6x] + 3 \operatorname{Cosh}[8x]) + \right. \\ \left. \frac{1}{320} b^3 (140 \operatorname{Cosh}[2x] + 100 \operatorname{Cosh}[4x] + 50 \operatorname{Cosh}[6x] + 15 \operatorname{Cosh}[8x] + 2 \operatorname{Cosh}[10x]) \right)$$

**Problem 1027: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cosh}[x] \operatorname{Sinh}[x]^3 (a + b \operatorname{Sinh}[x]^2)^3 dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{a (a + b \operatorname{Sinh}[x]^2)^4}{8 b^2} + \frac{(a + b \operatorname{Sinh}[x]^2)^5}{10 b^2}$$

Result (type 3, 114 leaves):

$$\frac{1}{10240} \left( -20 (64 a^3 + 24 a b^2 - 7 b^3) \operatorname{Cosh}[2x] + 20 (16 a^3 + 18 a b^2 - 5 b^3) \operatorname{Cosh}[4x] + \right. \\ \left. b (-10 (16 a - 5 b) b \operatorname{Cosh}[6x] + 15 (2 a - b) b \operatorname{Cosh}[8x] + 2 b^2 \operatorname{Cosh}[10x] + 320 ((-4 a + b)^2 - b^2 \operatorname{Cosh}[2x]) \operatorname{Sinh}[x]^6) \right)$$

**Problem 1052: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Cosh}[a + b x]^4 - \operatorname{Sinh}[a + b x]^4}{\operatorname{Cosh}[a + b x]^4 + \operatorname{Sinh}[a + b x]^4} dx$$

Optimal (type 3, 51 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[1 - \sqrt{2} \tanh[a + b x]\right]}{\sqrt{2} b} + \frac{\text{ArcTan}\left[1 + \sqrt{2} \tanh[a + b x]\right]}{\sqrt{2} b}$$

Result (type 7, 194 leaves):

$$-\frac{1}{2b} \left( \text{Cosh}[2a] \text{RootSum}\left[1 + 6e^{4a} \#1^2 + e^{8a} \#1^4, \frac{2bx - \text{Log}\left[e^{2bx} - \#1\right] + 2bx \#1^2 - \text{Log}\left[e^{2bx} - \#1\right] \#1^2}{3\#1 + e^{4a} \#1^3} \&\right] + \right. \\ \left. \text{RootSum}\left[1 + 6e^{4a} \#1^2 + e^{8a} \#1^4, \frac{-2bx + \text{Log}\left[e^{2bx} - \#1\right] + 2bx \#1^2 - \text{Log}\left[e^{2bx} - \#1\right] \#1^2}{3\#1 + e^{4a} \#1^3} \&\right] \text{Sinh}[2a] \right)$$

**Problem 1053: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cosh}[a + bx]^3 - \text{Sinh}[a + bx]^3}{\text{Cosh}[a + bx]^3 + \text{Sinh}[a + bx]^3} dx$$

Optimal (type 3, 47 leaves, 5 steps):

$$-\frac{4 \text{ArcTan}\left[\frac{1 - 2 \tanh[a + bx]}{\sqrt{3}}\right]}{3\sqrt{3} b} - \frac{1}{3b(1 + \tanh[a + bx])}$$

Result (type 3, 115 leaves):

$$\frac{1}{18b} (-\text{Cosh}[a + bx] + \text{Sinh}[a + bx]) \left( \left( 3 + 8\sqrt{3} \text{ArcTan}\left[\frac{\text{Sech}[bx] (\text{Cosh}[2a + bx] - 2\text{Sinh}[2a + bx])}{\sqrt{3}}\right] \right) \text{Cosh}[a + bx] + \right. \\ \left. \left( -3 + 8\sqrt{3} \text{ArcTan}\left[\frac{\text{Sech}[bx] (\text{Cosh}[2a + bx] - 2\text{Sinh}[2a + bx])}{\sqrt{3}}\right] \right) \text{Sinh}[a + bx] \right)$$

**Problem 1055: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cosh}[a + bx] - \text{Sinh}[a + bx]}{\text{Cosh}[a + bx] + \text{Sinh}[a + bx]} dx$$

Optimal (type 3, 22 leaves, 1 step):

$$-\frac{1}{2b(\text{Cosh}[a + bx] + \text{Sinh}[a + bx])^2}$$

Result (type 3, 65 leaves):

$$-\frac{\text{Cosh}[2a] \text{Cosh}[2bx]}{2b} + \frac{\text{Cosh}[2bx] \text{Sinh}[2a]}{2b} + \frac{\text{Cosh}[2a] \text{Sinh}[2bx]}{2b} - \frac{\text{Sinh}[2a] \text{Sinh}[2bx]}{2b}$$

**Problem 1056: Result more than twice size of optimal antiderivative.**

$$\int \frac{-\text{Csch}[a+bx] + \text{Sech}[a+bx]}{\text{Csch}[a+bx] + \text{Sech}[a+bx]} dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$\frac{1}{b(1 + \text{Tanh}[a+bx])}$$

Result (type 3, 65 leaves):

$$\frac{\text{Cosh}[2a] \text{Cosh}[2bx]}{2b} - \frac{\text{Cosh}[2bx] \text{Sinh}[2a]}{2b} - \frac{\text{Cosh}[2a] \text{Sinh}[2bx]}{2b} + \frac{\text{Sinh}[2a] \text{Sinh}[2bx]}{2b}$$

**Problem 1059: Result is not expressed in closed-form.**

$$\int \frac{-\text{Csch}[a+bx]^4 + \text{Sech}[a+bx]^4}{\text{Csch}[a+bx]^4 + \text{Sech}[a+bx]^4} dx$$

Optimal (type 3, 51 leaves, 6 steps):

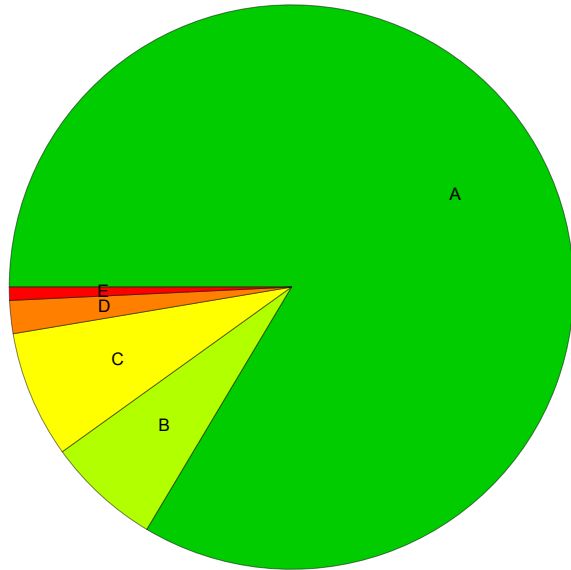
$$\frac{\text{ArcTan}[1 - \sqrt{2} \text{Tanh}[a+bx]]}{\sqrt{2}b} - \frac{\text{ArcTan}[1 + \sqrt{2} \text{Tanh}[a+bx]]}{\sqrt{2}b}$$

Result (type 7, 194 leaves):

$$\frac{1}{2b} \left( \text{Cosh}[2a] \text{RootSum}\left[1 + 6e^{4a} \#1^2 + e^{8a} \#1^4 \&, \frac{2bx - \text{Log}[e^{2bx} - \#1] + 2bx \#1^2 - \text{Log}[e^{2bx} - \#1] \#1^2}{3\#1 + e^{4a} \#1^3} \&\right] + \right. \\ \left. \text{RootSum}\left[1 + 6e^{4a} \#1^2 + e^{8a} \#1^4 \&, \frac{-2bx + \text{Log}[e^{2bx} - \#1] + 2bx \#1^2 - \text{Log}[e^{2bx} - \#1] \#1^2}{3\#1 + e^{4a} \#1^3} \&\right] \text{Sinh}[2a] \right)$$

## Summary of Integration Test Results

1059 integration problems



A - 885 optimal antiderivatives

B - 69 more than twice size of optimal antiderivatives

C - 77 unnecessarily complex antiderivatives

D - 20 unable to integrate problems

E - 8 integration timeouts